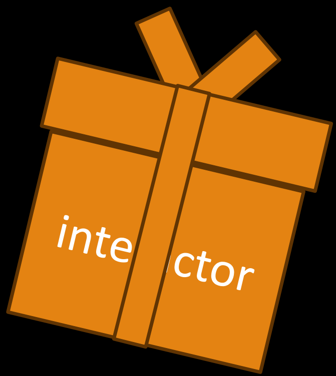
Commentaries on Problems

JUDGE TEAM

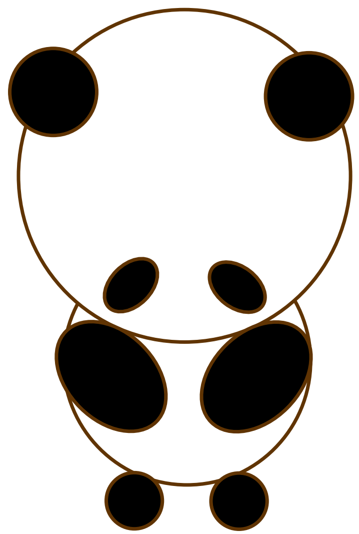
ICPC 2023 2024 ASIA YOKOHAMA REGIONAL

BLACK FRIDAY

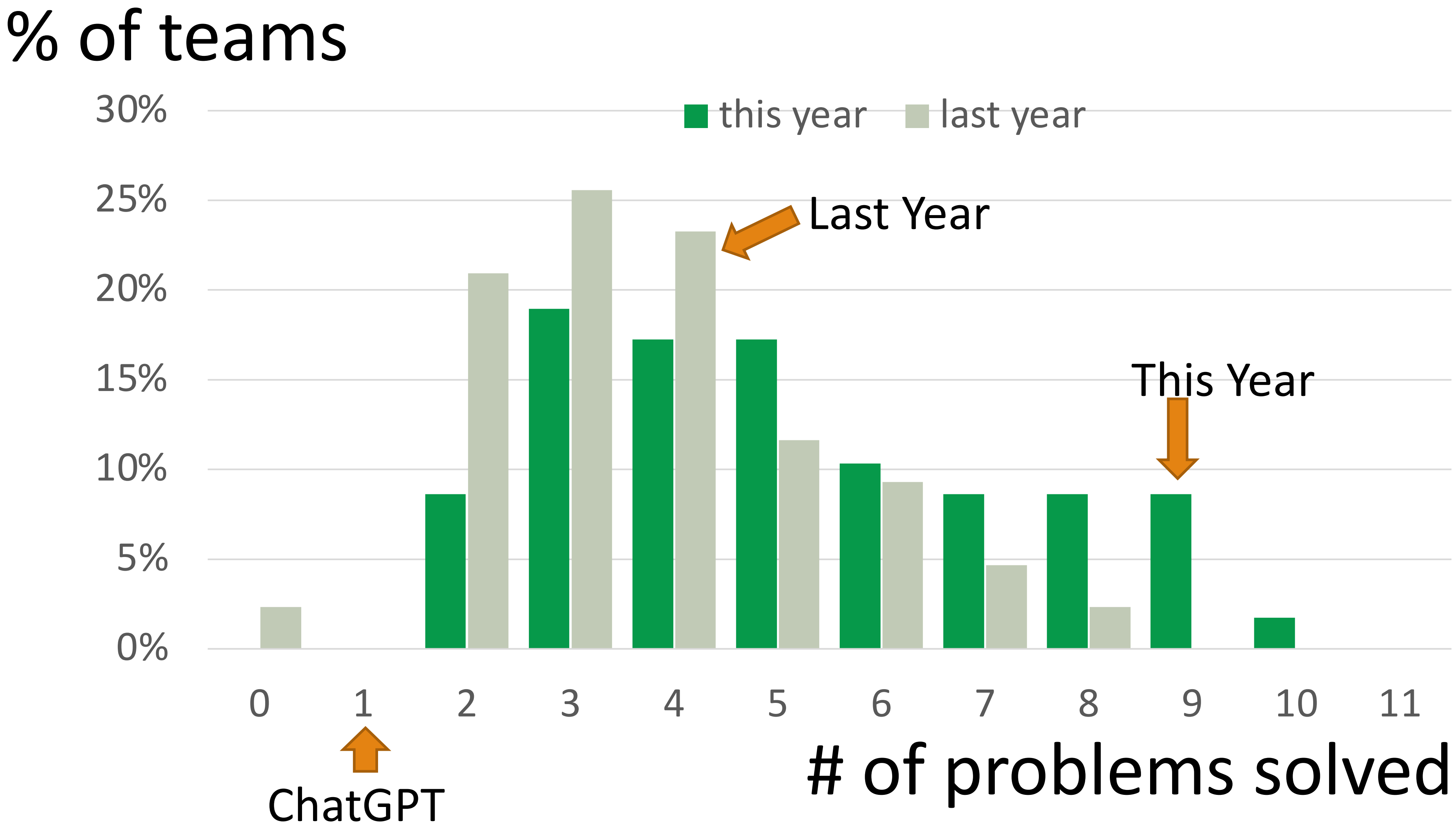


discount difficulties

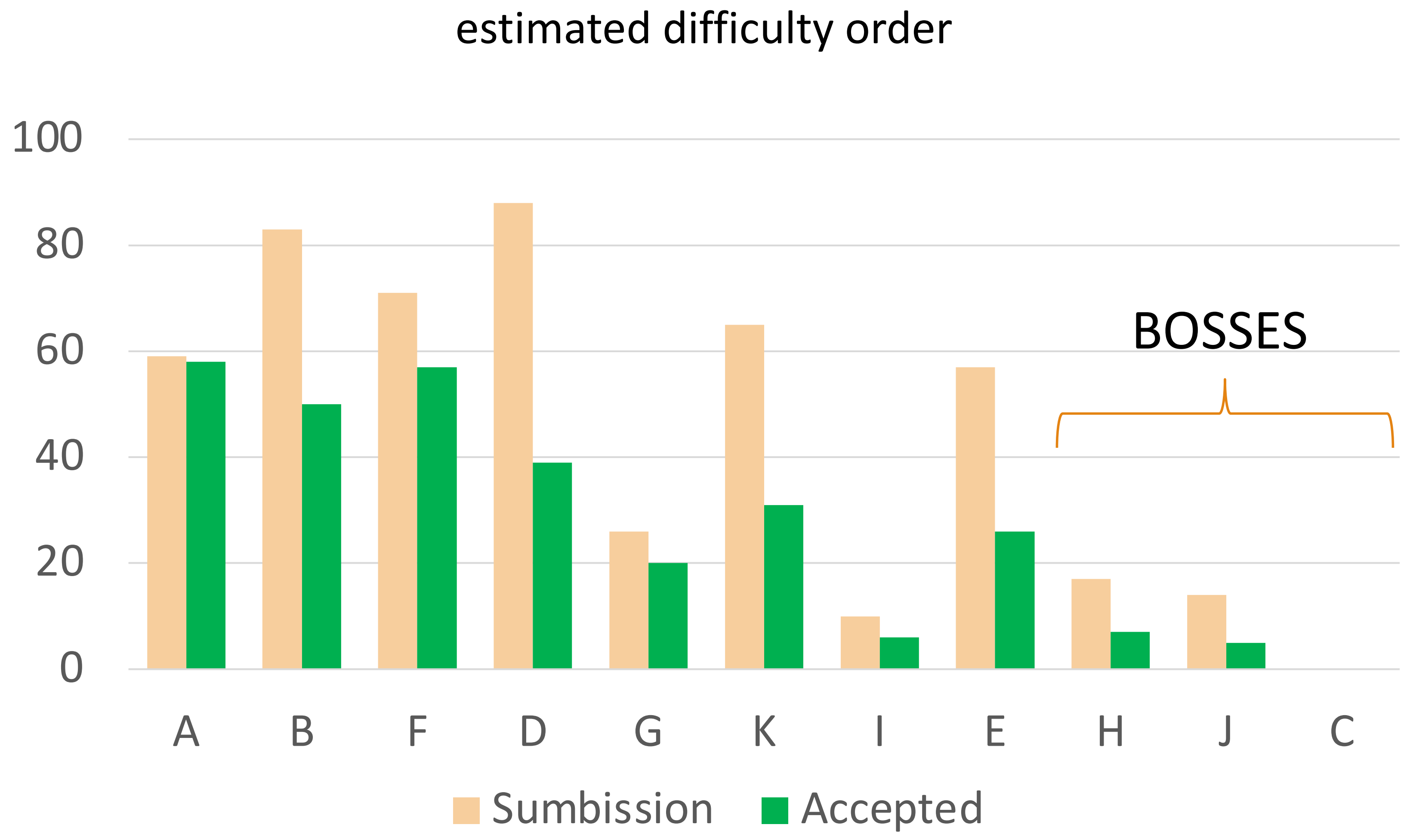
Sorry about the accident …



Solved vs. Teams @Freeze



Problem vs. #Teams @Freeze



A: Yokohama Phenomena

PROPOSER: KAZUHIRO INABA
AUTHOR: TOMOHARU UGAWA

Problem Description

Count “YOKOHAMA” hidden in the board

Y	O	H	A
O	K	A	M

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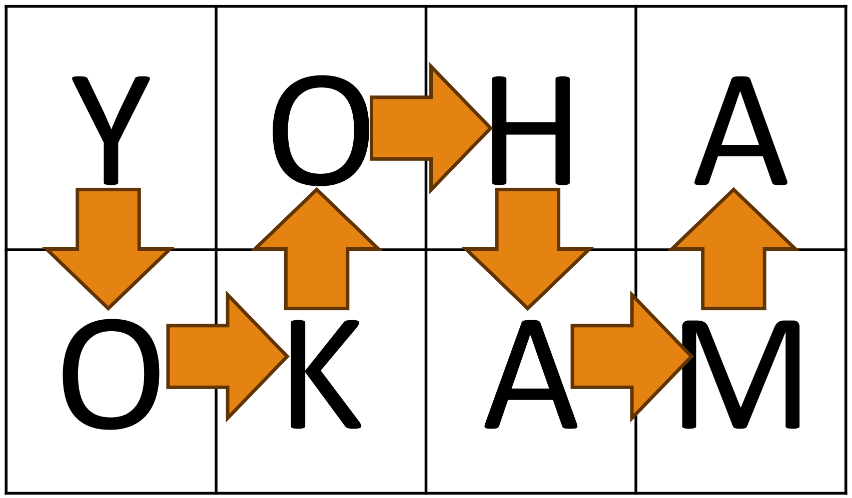
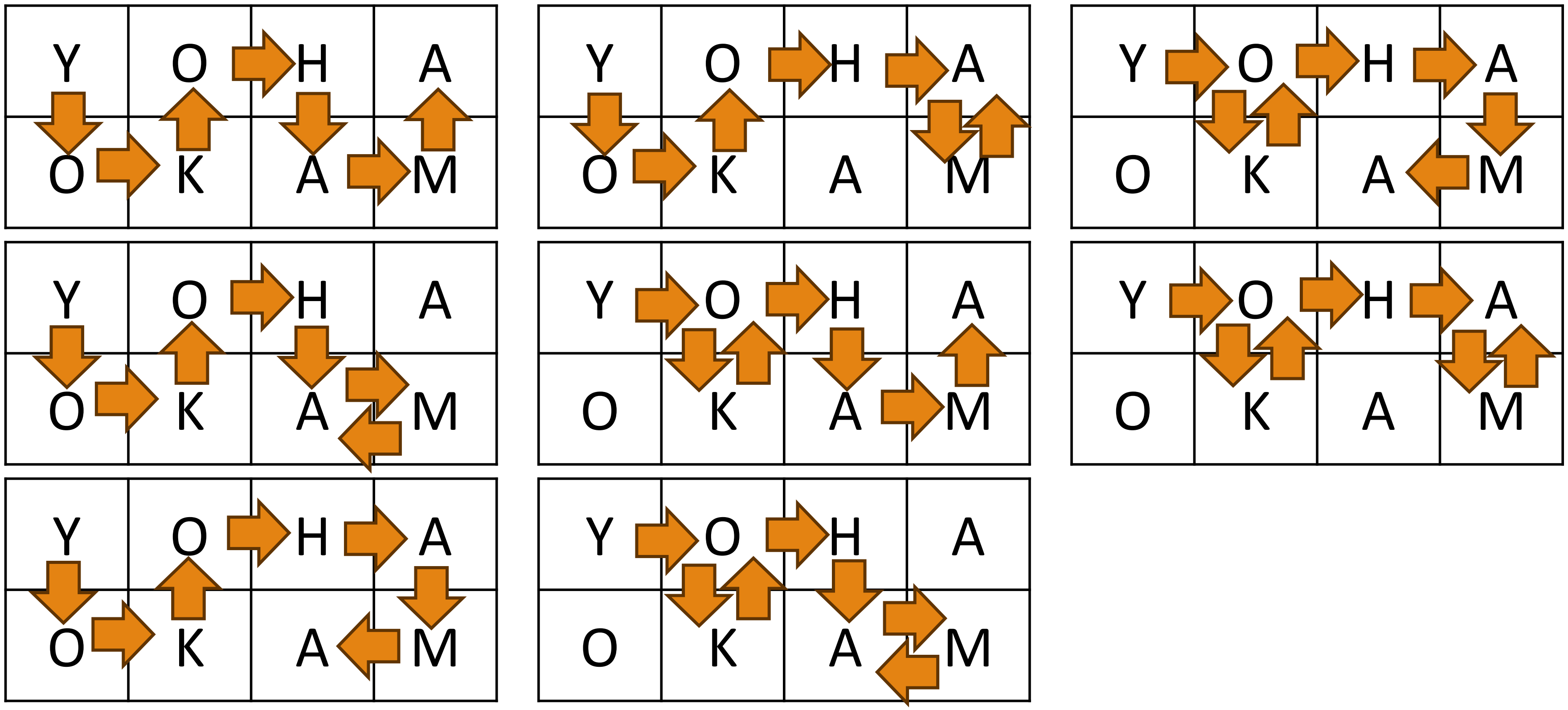


Problem Description

Count “YOKOHAMA” hidden in the board



Any enumeration will work

- depth-first search
- dynamic programming

Y	O	H	A
O	K	A	M

1			

Y

1	1		
1			

O

	1		
1	2		

K

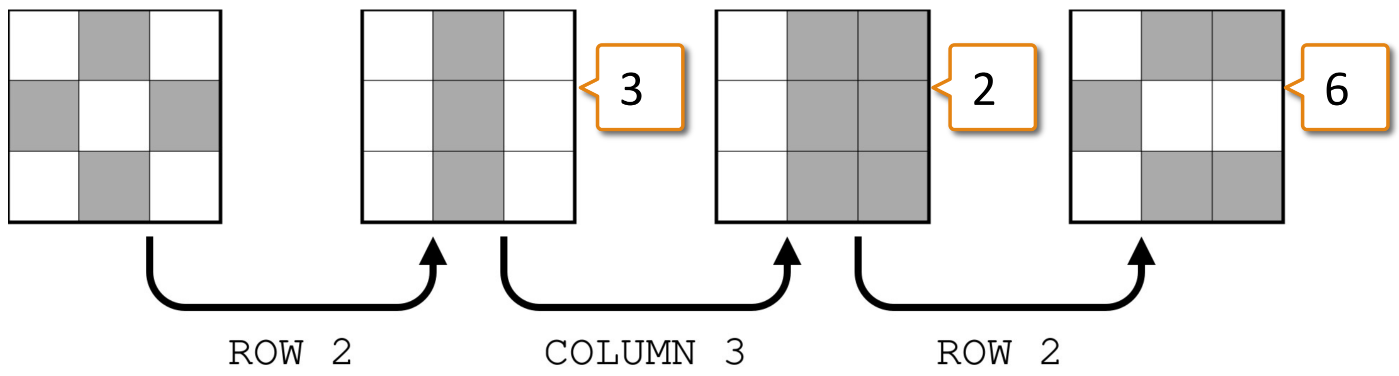
…

F: Color Inversion on a Huge Chessboard

PROPOSER: KOHEI MORITA
AUTHOR: KOHEI MORITA

Problem

- Given $N, Q (1 \leq N, Q \leq 500{,}000)$ (as usual)
- You have to process Q queries for $N \times N$ chessboard.
  - Flip color of a row
  - Flip color of a column
- Print # of areas (= same color components) after each query



Key Point

- You can notice that each area forms rectangle.
  - Let’s try with a random case.

```
bash-3.2$ ./a.out
Random test with n = 20 / q = 100
#.#####.##..#.###..#
#.#####.##..#.###..#
.#.....#..##.#...##.
#.#####.##..#.###..#
.#.....#..##.#...##.
#.#####.##..#.###..#
#.#####.##..#.###..#
#.#####.##..#.###..#
#.#####.##..#.###..#
#.#####.##..#.###..#
.#.....#..##.#...##.
.#.....#..##.#...##.
#.#####.##..#.###..#
#.#####.##..#.###..#
.#.....#..##.#...##.
.#.....#..##.#...##.
#.#####.##..#.###..#
.#.....#..##.#...##.
.#.....#..##.#...##.
.#.....#..##.#...##.
```

  - Why: row-i color is same with row-1 or inversion of row-1

Solution

- Managing row-1 color & column-1 color.
  - And, (# of connected component) of row-1 & column-1.
- Print (# of area of row-1) * (# of area of column-1) after query
- You can process each query in $O(1)$ time, total time complexity is $O(N+Q)$

B: Rank Promotion

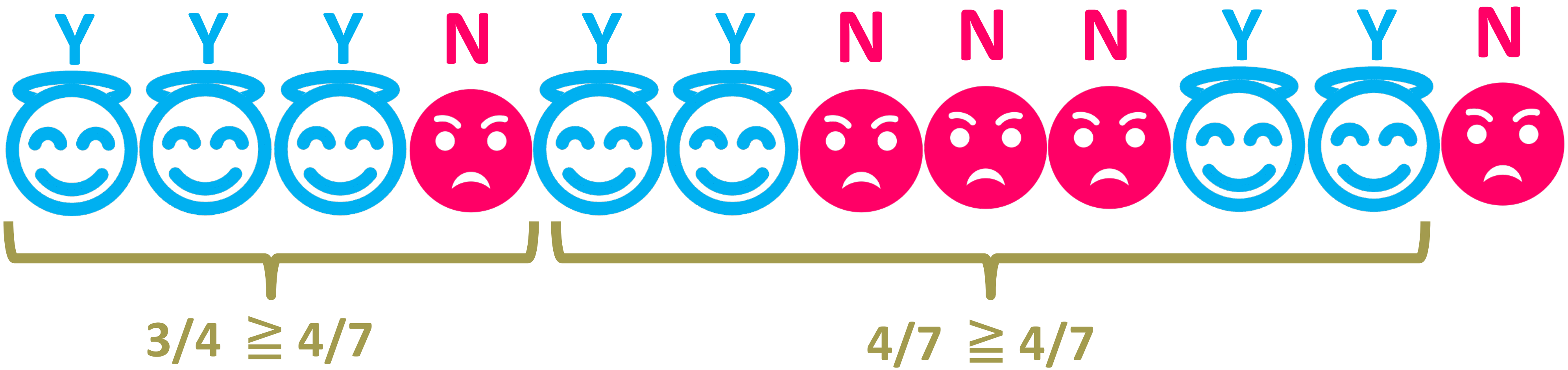
PROPOSER: KAZUHIRO INABA
AUTHOR: KAZUHIRO INABA

$n \leqq 500000$
$c \leqq 200$

Problem

If a sufficiently long (≧c) range contains Y's in a sufficiently high (≧p/q) ratio, rank += 1.
What's the final rank?

Sample Input: c=4, p/q = 4/7



Solution: O(nc)

No need to think about too-long (≧ 2c) ranges. Just check the Y-ratio of all the len ≦ 2c-1 substrings.

If a 2c sequence has a high Y-ratio,

ratio(Y) ≧ p/q

either the first or the latter half also has.

ratio(Y) ≧ p/q

or

ratio(Y) ≧ p/q

Advanced Solution : O(n)

You can solve the problem even if the upperbound of c were large.

$$\frac{\sum_{i=1}^{k} x_i}{k} \geq r$$

Average is larger than r.

$$\Updownarrow$$

$$\sum_{i=1}^{k} (x_i - r) \geq 0$$

Sum of x_i-r is above 0.

Maintain the cumulative sum of **`(S[i]=='Y'?1:0)-p/q`** and the max after the last rank promotion. Then, in O(1) you can check if a “higher than p/q” range exists.

D: Nested Repetition Compression

PROPOSER: KENTO EMOTO
AUTHOR: TAKASHI CHIKAYAMA

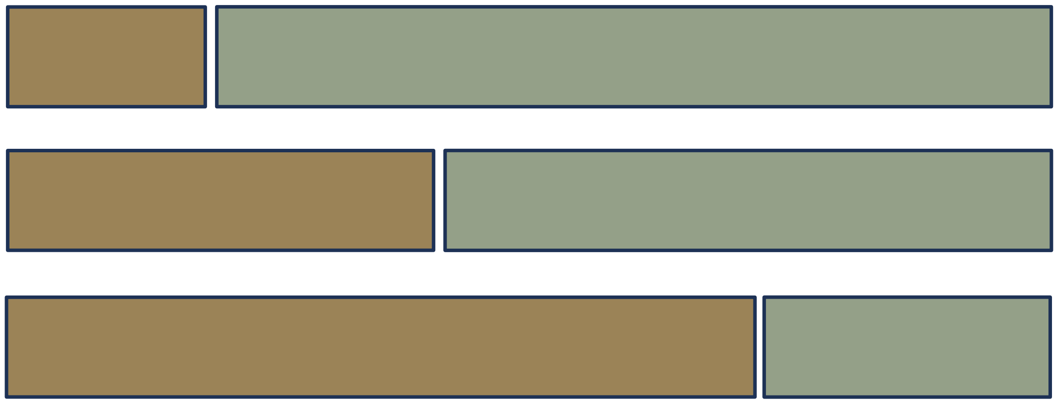
Compression Specifying Repetitions

- Up to nine repetitions of the same string can be specified
  - ababab → 3(ab)
  - abababaaaaa → 3(ab)5(a)
- Repetitions can be arbitrarily nested
  - aaaaaaaaaaaa → 3(4(a))
- As this compression scheme is *context-free,* compression of distinct substrings are *independent*

The Best Compression is Either:

- Repetition of optimally compressed segments,

- Two optimally compressed ones concatenated, or



- As is, i.e., no compression at all.

Preparation: Repetition Table

For all the segments beginning from all the positions in the original string, a table of repeated patterns and their lengths should be prepared.

The table can be made with complexity $O(n^3)$.

Bottom-up Construction

Build a table of the shortest representations for all the string segments, starting from the shortest ones and gradually expanding to longer ones.

- Any segments of length four or less should be *as-is.*
- Knowing the shortest reps for lengths n and less, the shortest for of length $n+1$ segments are either:
  - Concatenation of the shortest reps of the first k characters and the remaining $n+1-k$ characters, for $k = 1, \ldots, n$. This can be checked with complexity of $O(n)$, or
  - Repetition of j identical segments of length $(n+1)/j$ for any factor j of $n+1$. Whether this is possible can be looked up in the repetition table.

The total complexity is $O(n^3)$.

K: Probing the Disk

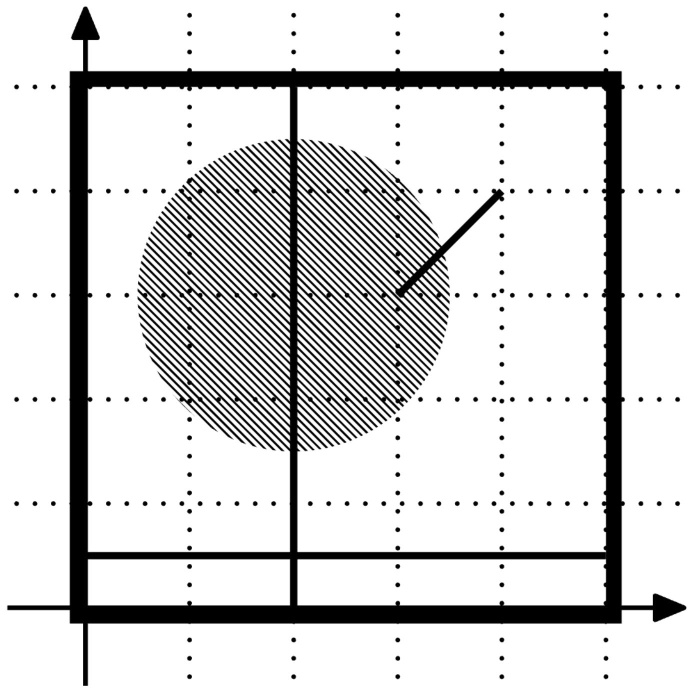
PROPOSER: KIMINORI MATSUZAKI
AUTHOR: KIMINORI MATSUZAKI
MITSURU KUSUMOTO

Problem

Given a disk (radius $\geq$ 100) in a square (side = 10^5), decide the position and the size of the disk, by at most 1024 probes.

Each prove:

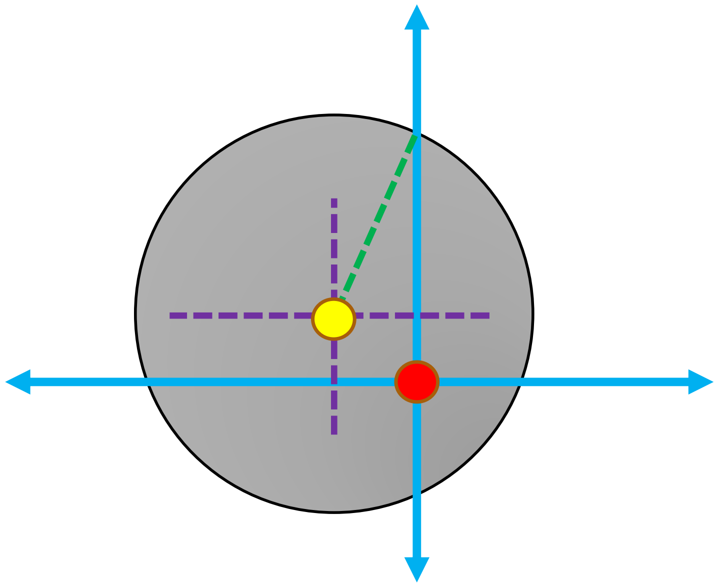
- Query: a line segment
- Answer: length on disk



Key to Solution

"Find a point that is surely in a disk"

If you find a point in a disk, you can solve the problem in 4 more probes.

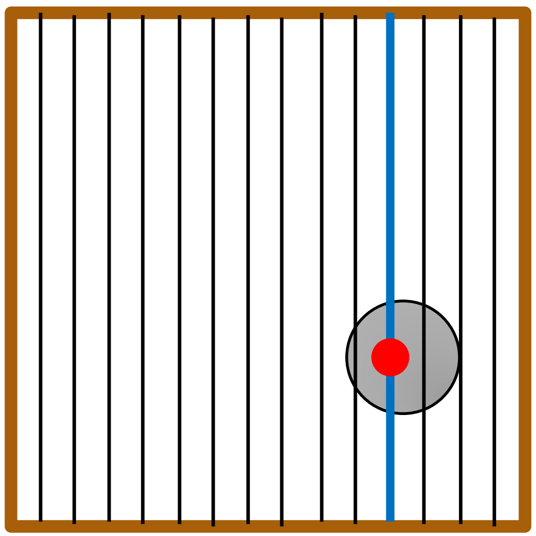


A Simple Solution

1. Probe by vertical lines (1000 probes)
   and find a line with the largest common length

2. Do binary search (11 probes)
   to find a point that is surely in the disk

3. Find the center
   and radius (4 probes)



E: Chayas

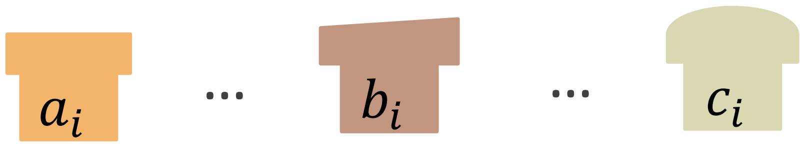
PROPOSER: SOU KUMABE
AUTHOR: SHINYA SHIROSHITA

Overview

There were n chayas (teahouses) in a line.

You have m records showing the following information:

Record i: chaya b_i is between chaya a_i and c_i.

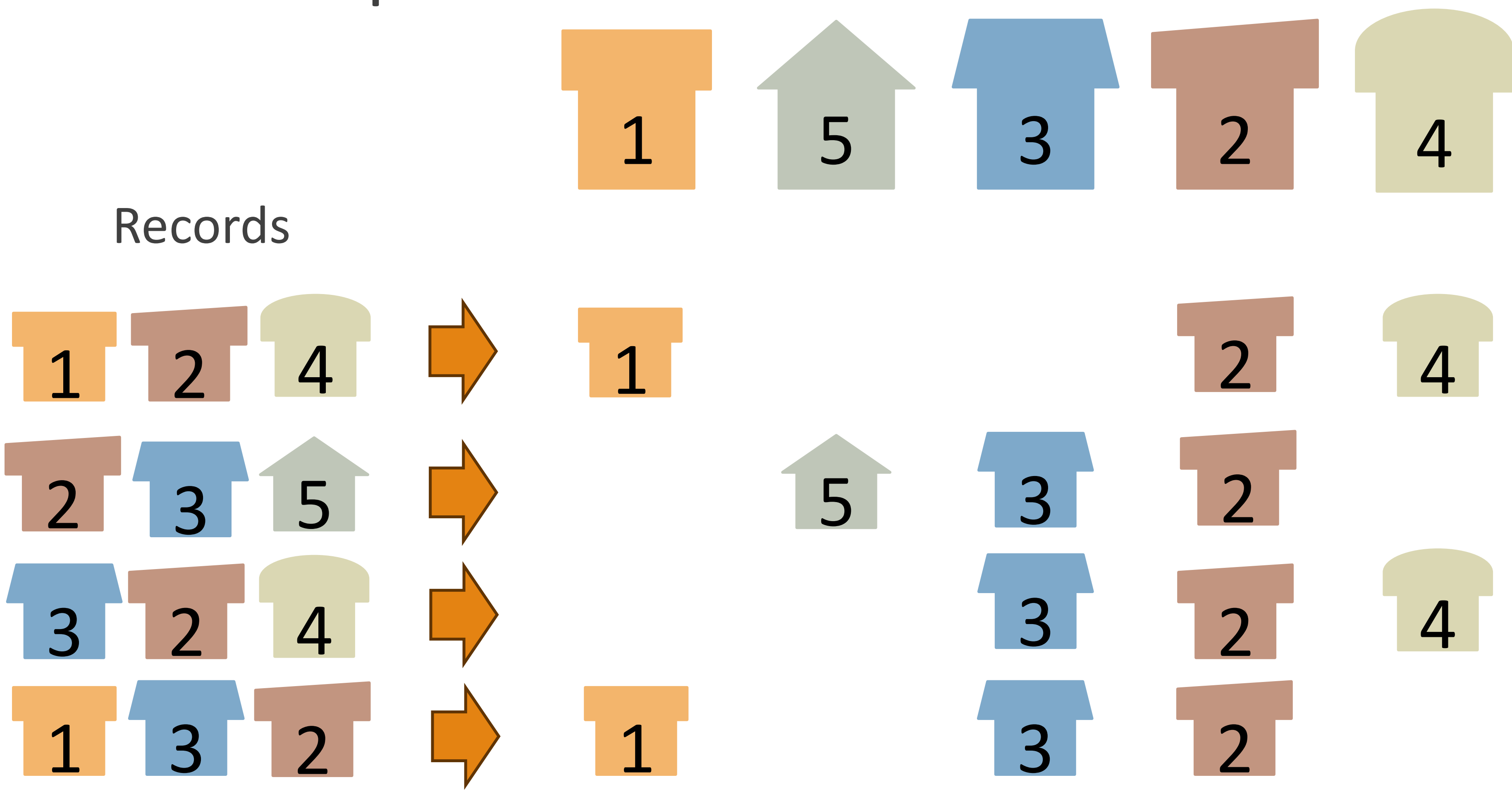


※ Reversing the order is OK

How many orders were there satisfying all the records?

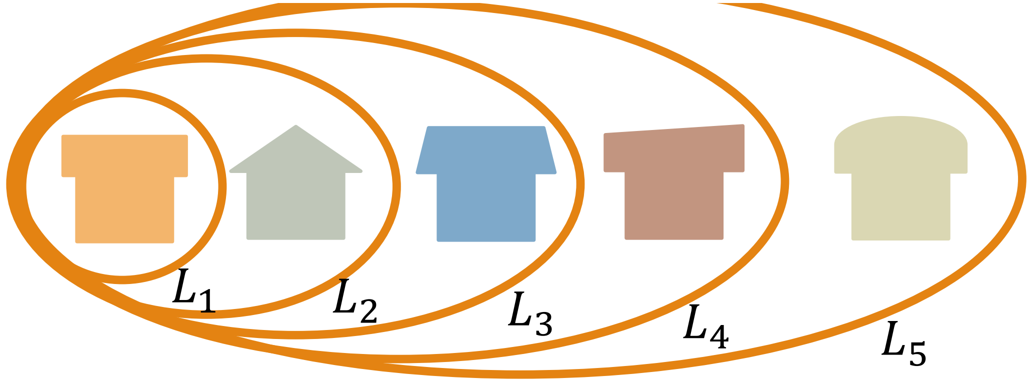
- $3 \leq n \leq 24$
- $1 \leq m \leq n(n-1)(n-2)/2$

Example



※ Reversing the order is OK

Analysis

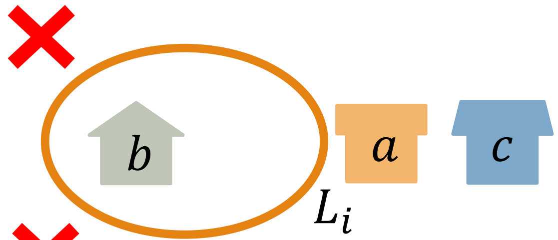


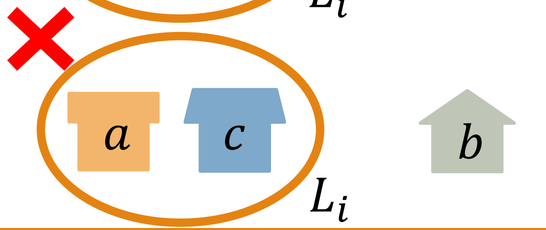
Let's consider when we select chayas from left to right.

Let L_i be the subset of the i chayas from the left.

The condition "b is between a and c" can be formulated as follows:

- For all $1 \leq i \leq n-1$, **NONE** of the below must hold.
  - $b \in L_i$ and none of a, c are not in L_i.
  - $b \notin L_i$ and both of a, c are in L_i.





How to check these conditions quickly?

Analysis

For simplicity, we hereby consider the condition

$b \in S$ and none of a, c are not in S

$= S$ where $\{b\} \subseteq S \subseteq (\text{all chayas}) \setminus \{a, c\}$

for each of the records.

When we create a 2^n boolean tables memorizing the conditions' true or false, the naïve check for each record takes $O(m \cdot 2^n) = \ O(n^3 \cdot 2^n)$, which is too slow.

→ *Let's focus on all the records whose b are the same.*

Precomputation

When we define

$$f(S) = \begin{cases} 1 \text{ if } S = (\text{all chayas}) \setminus \{a_i, c_i\} \text{ for some } (a_i, b_i, c_i), \\ 0 \text{ otherwise,} \end{cases}$$

Then, the subset S containing b contradicts the records if

$$g(S) := \sum_{S \subseteq T} f(T) \geq 1.$$

where $g(S)$ is the sum of fs of the supersets of S.

The calculation of $g(S)$ can be speed up based on **Fast Zeta Transformation**.

Precomputation

The following dp calculates $g(S) = \mathrm{dp}[n][S]$.

$\mathrm{dp}[0][S] = f(S)$ **for each subset** S.
for each chaya $i = 1, \dots, n$:
  for each subset S **of** $i \notin S$:
    $\mathrm{dp}[i][S] = \mathrm{dp}[i-1][S] + \mathrm{dp}[i-1][S \cup \{i\}]$
  for each subset S **of** $i \in S$:
    $\mathrm{dp}[i][S] = \mathrm{dp}[i-1][S]$

S	.	3	2	23	1	13	12	123
dp[0]	0	0	0	1	0	0	1	0
dp[1]	0	0	1	1	0	0	1	0
dp[2]	1	1	1	1	1	0	1	0
dp[3]	1	1	1	1	1	0	1	0

An example where chayas are $\{1, 2, 3, b\}$ and queries are $(1, b, 2)$ and $(2, b, 3)$.

Precomputation

The transformation of the same b can be done in $O(n \cdot 2^n)$.

For other bs, we can calculate in the same transformation when we use different digits, totaling $O(n \cdot 2^n)$ time complexity.

We can solve the other condition in the similar way.

Solution

The solution is equal to the number of sequences where the left i chayas satisfies the records for all $0 \leq i \leq n$.

This can be solved by dynamic programming.

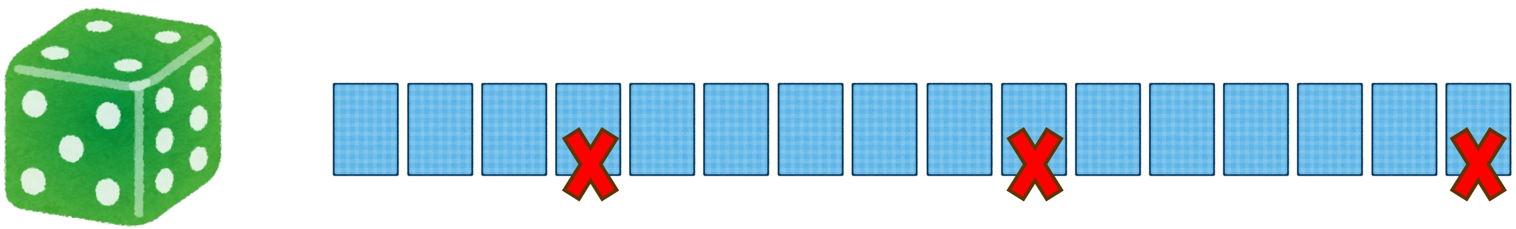
As the condition check of each subset takes $O(1)$ by the precomputation, the time complexity is $O(n \cdot 2^n)$.

G: Fortune Telling

PROPOSER: MITSURU KUSUMOTO
AUTHOR: MITSURU KUSUMOTO

Problem Overview

- n cards are lined up ($2 \leq n \leq 300000$)
- Each time, we roll a die and when it shows x, we remove cards x-th, $(x+6)$-th, $(x+12)$-th, … from left.
- We end this when only one card remains.
- Compute the probability each initial card survives.



Naive DP

dp[n'][k] := "Probability that, when there are
n' cards, card k-th from left survives"

Naive DP

dp[n'][k] := "Probability that, when there are n' cards, card k-th from left survives"

$\Theta(n^2)$ entries!! Too many!! 😭

Dependency

dp[n][:]

Dependency

dp[n][:] **only** depends on

dp[(5/6)n][:]

dp[(5/6)n+1][:]

Dependency

dp[n][:] **only** depends on

dp[$(5/6)n$][:]

dp[$(5/6)n+1$][:] **only** depend on

dp[$(5/6)^2n$][:]

dp[$(5/6)^2n+1$][:]

dp[$(5/6)^2n+2$][:]

Dependency

dp[n][:] **only** depends on

dp[$(5/6)n$][:]
dp[$(5/6)n+1$][:] **only** depend on

dp[$(5/6)^2n$][:]

Required entries for DP calculation may be **much smaller than n^2** ?

Bound

The number of required entries for DP computation is roughly bounded by

$$n \sum_{k=1}^{\infty} k \left(\frac{5}{6}\right)^{k-1}$$

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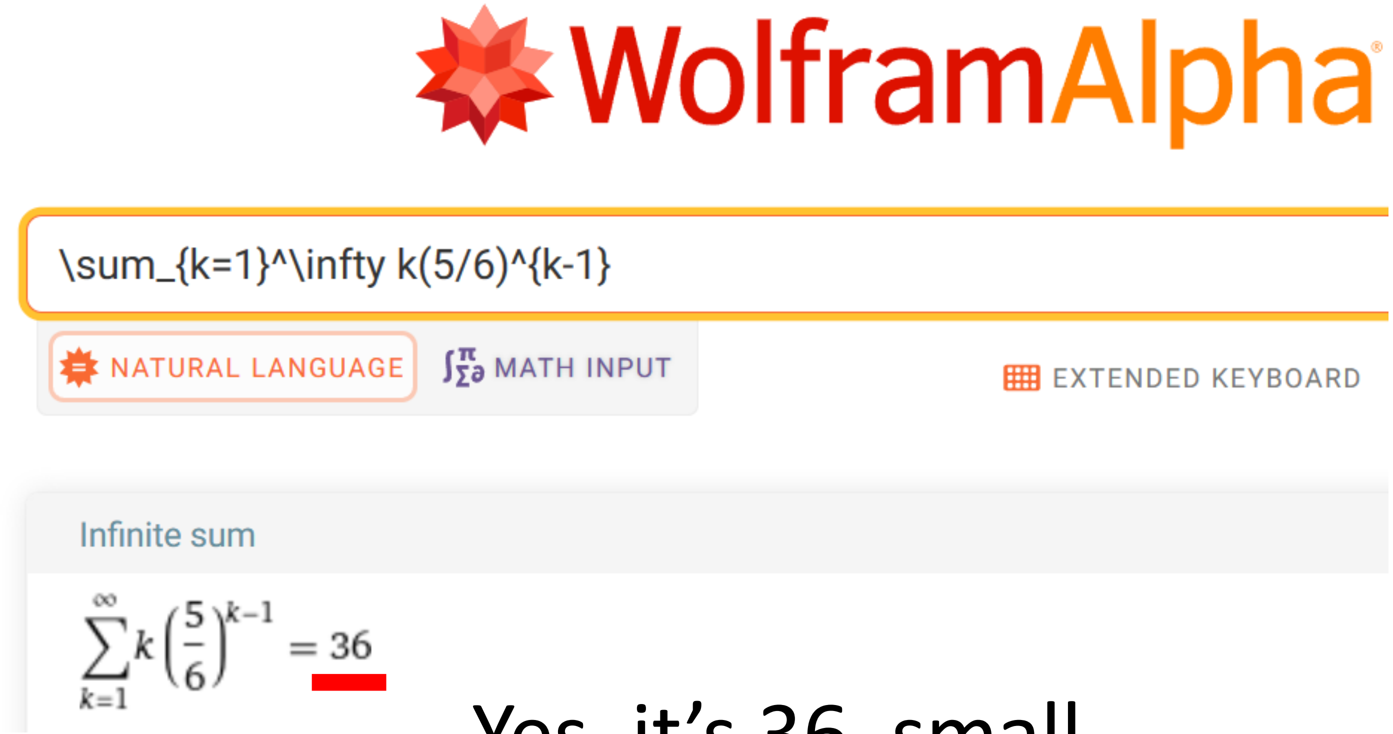
?

If you can access to Wolfram Alpha…



Yes, it's 36, small.

Another method

You can estimate it without Wolfram Alpha:

- Approximate it by a tiny code
- Differentiate $1+x+x^2+\ldots+x^n = (1-x^{n+1})/(1-x)$ and set $x=5/6$, then take $n\to\infty$.

$$n \sum_{k=1}^{\infty} k \left(\frac{5}{6}\right)^{k-1} = 36n$$

Solution

Compute a DP table with memorization.

In general, if the die has A faces, time complexity is $O(A^3n)$.

H: Task Assignment to Two Employees

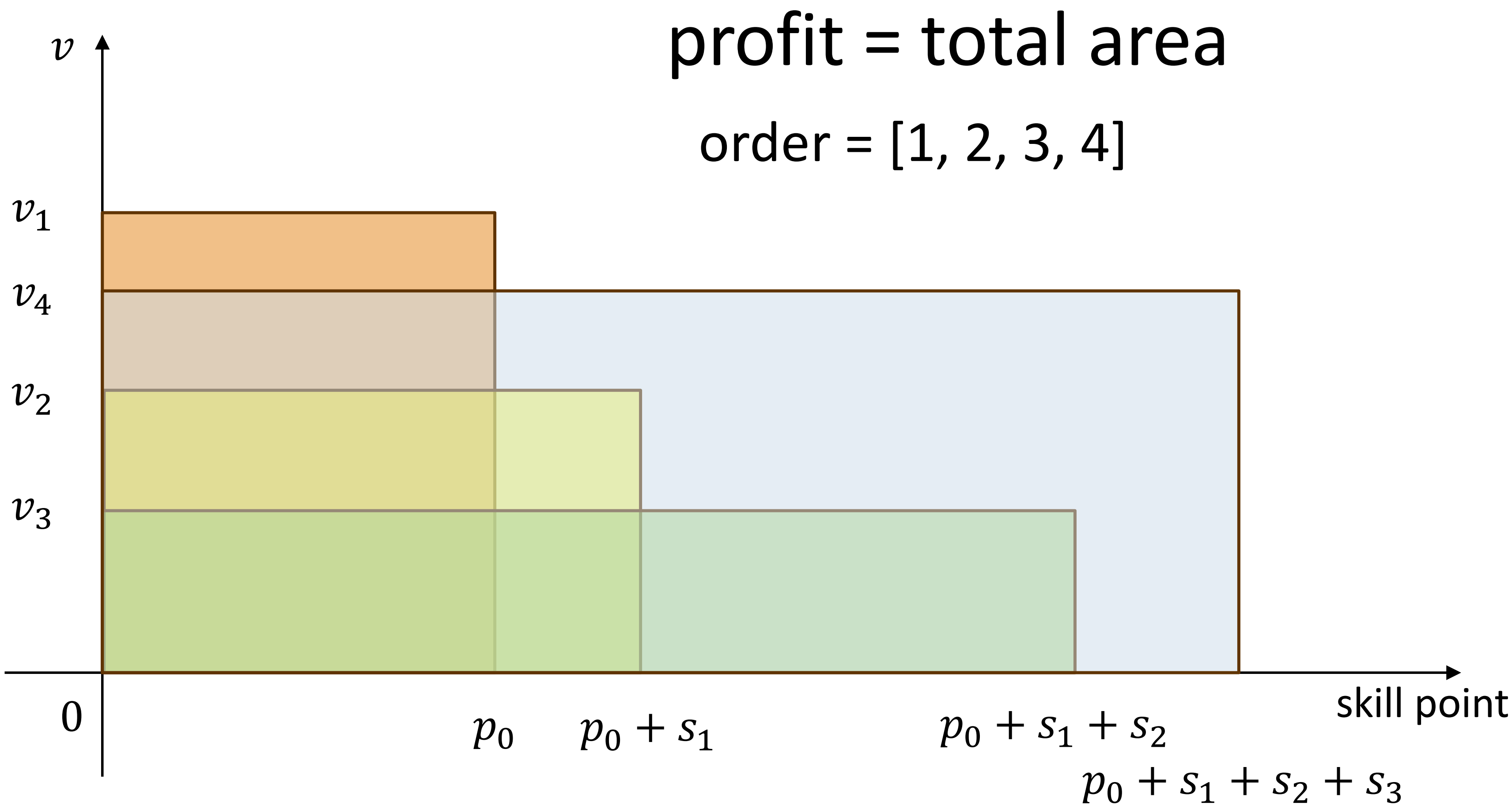
PROPOSER: YOICHI IWATA
AUTHOR: YOICHI IWATA

Problem

Assign tasks to two employees in an appropriate order to maximize the total profit.

- initial skill point: p_0
- task compatibility: $v_{i,j}$
- skill growth: $s_{i,j}$
- profit = current skill point $\times$ $v_{i,j}$
- new skill point = current skill point $+$ $s_{i,j}$

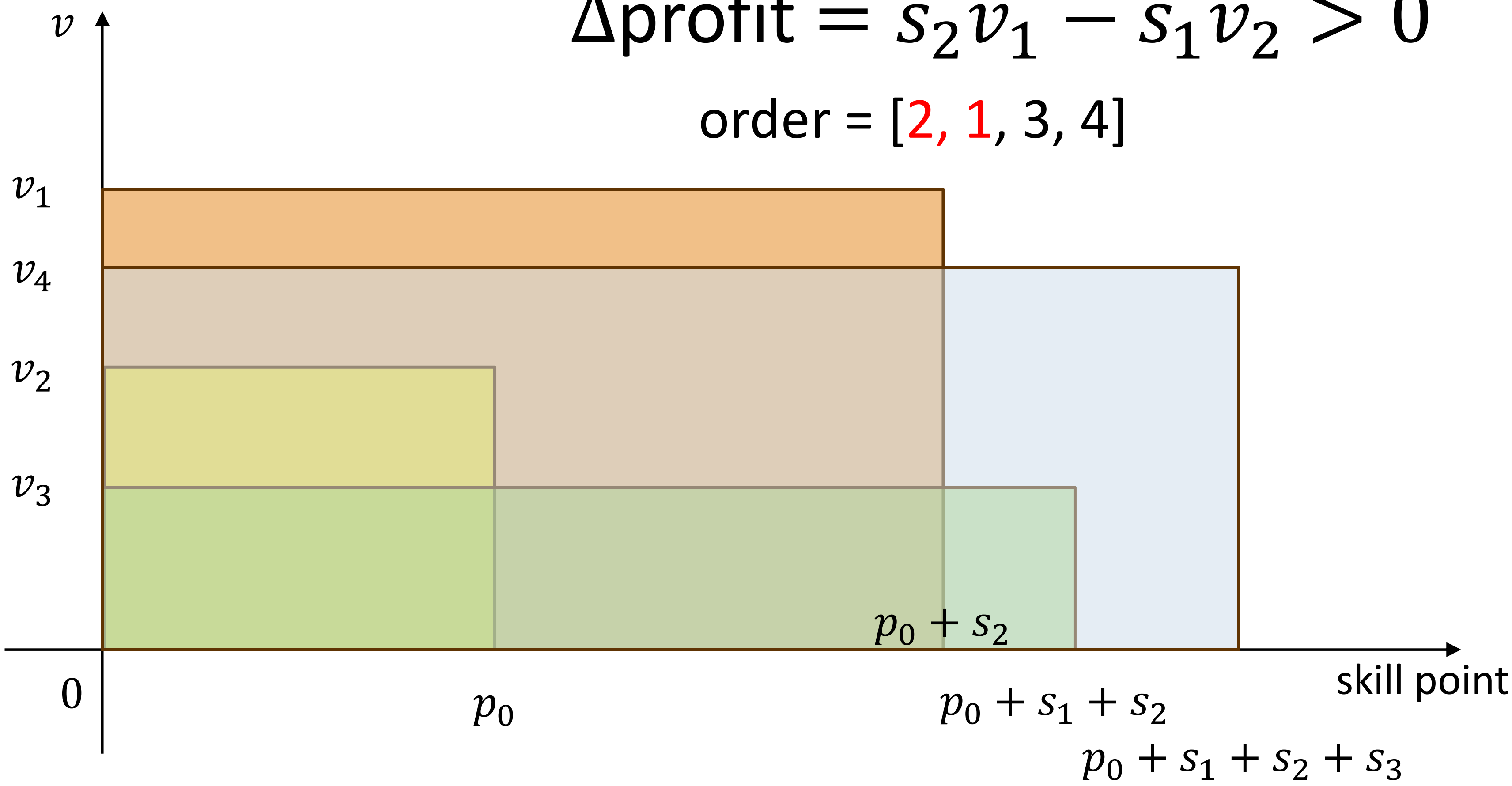
Optimize Ordering for Single Employee



Optimize Ordering for Single Employee

$$\Delta\text{profit} = s_2 v_1 - s_1 v_2 > 0$$

order = [2, 1, 3, 4]



Optimize Ordering for Single Employee

Optimal ordering $= [i_1, i_2, \ldots, i_n]$
s.t. $s_{i_{j+1}} v_{i_j} \leq s_{i_j} v_{i_{j+1}}$

$\Rightarrow$ Sort & Greedy

Key Observation

Optimal profit

$$= \sum_i p_0 v_i + \sum_{i,j} \max(s_i v_j, s_j v_i)$$

Optimize Assignment

x_i: task i is assigned to employee 1

Profit $=$

$$\sum_i p_0 v_{1,i} x_i + \sum_{i,j} \max(s_{1,i} v_{1,j}, s_{1,j} v_{1,i})\, x_i x_j$$
$$+ \sum_i p_0 v_{2,i} \bar{x}_i + \sum_{i,j} \max(s_{2,i} v_{2,j}, s_{2,j} v_{2,i})\, \overline{x_i} \overline{x_j}$$

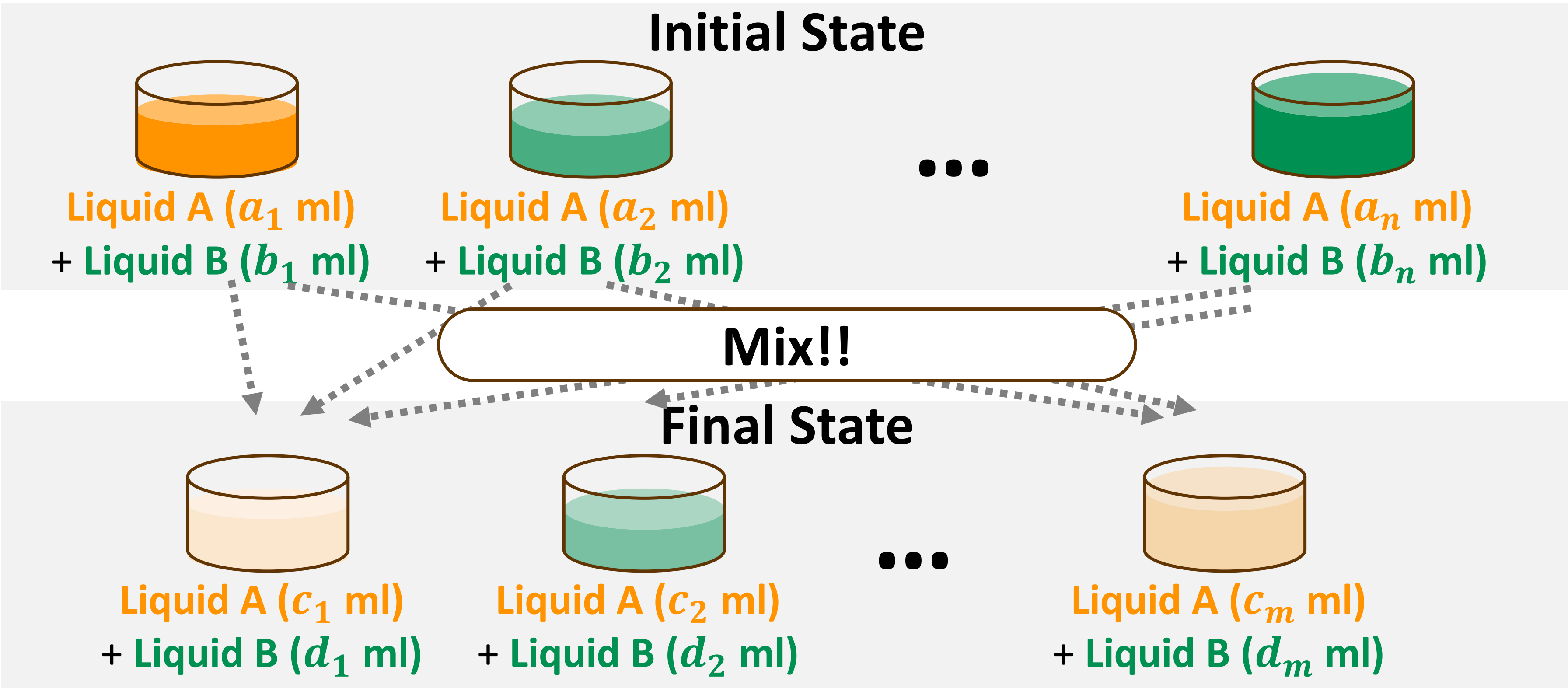
maximization of Quadratic pseudo-Boolean supermodular function → mincut !!!

I: Liquid Distribution

PROPOSER: RYOTARO SATO
AUTHOR: RYOTARO SATO

Problem Overview

Judge whether mixture process below is feasible.

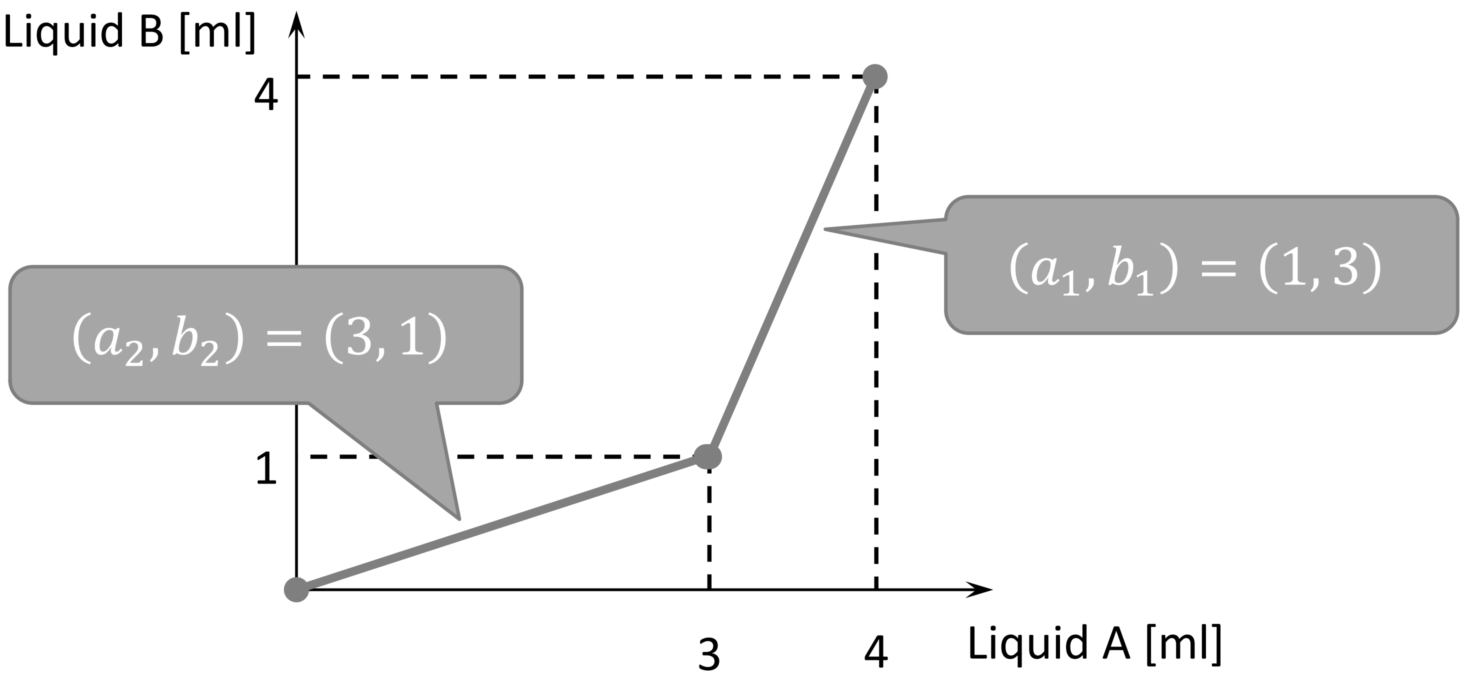


Constraints: $1 \leq n, m \leq 500$, $\sum a_i = \sum c_j$, $\sum b_i = \sum d_j$.

Observation: Curves

Sort all liquids by b_i/a_i (or d_j/c_j) and plot cumulative sum.

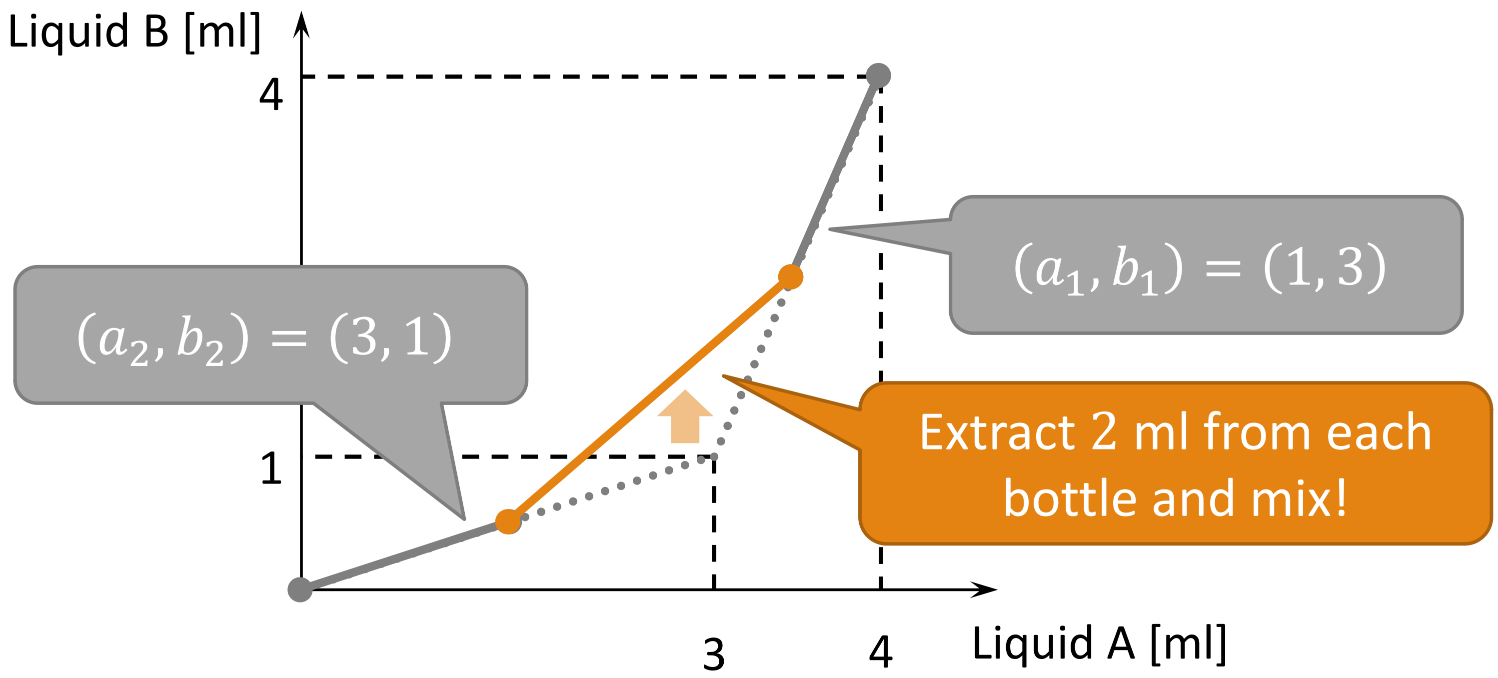
Generated curves are always convex.



Observation: Mixture

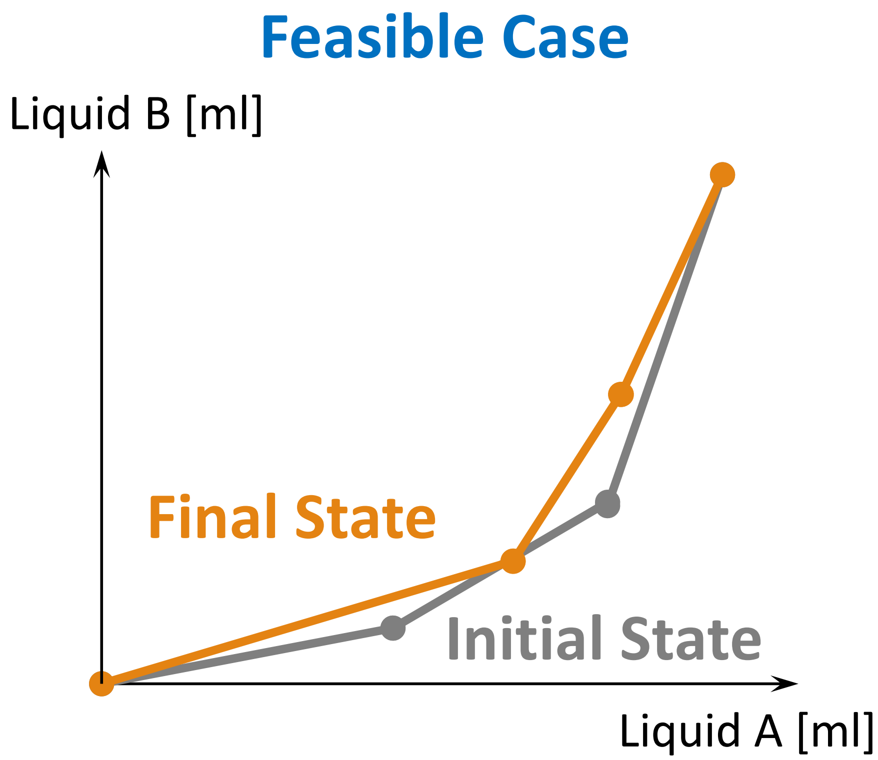
What happens to curves when liquids are mixed?

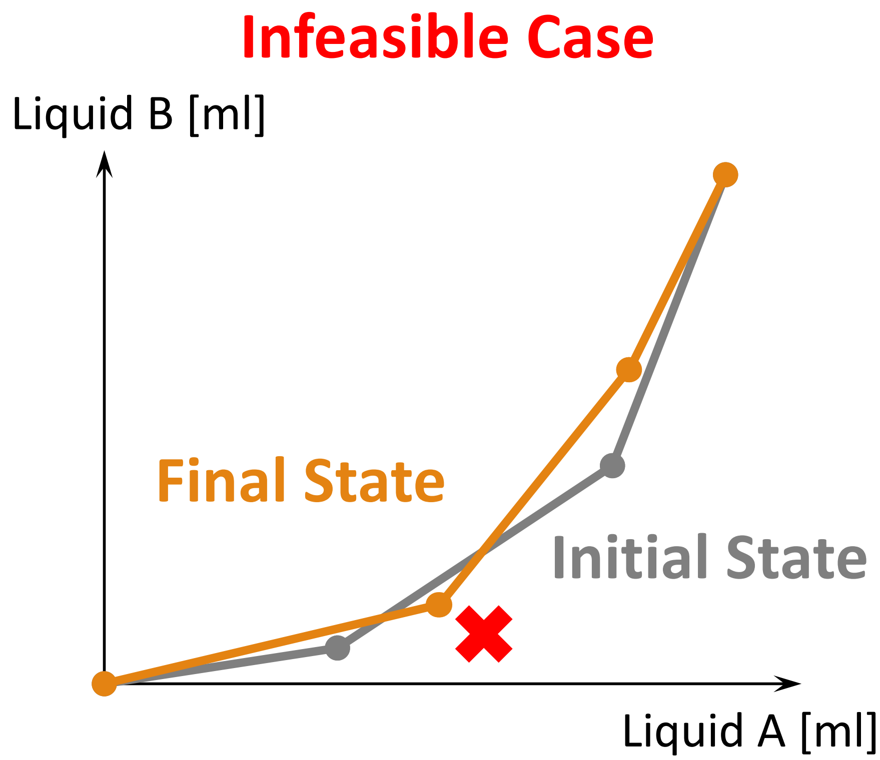
→ Curves **always move upper!**



Solution

Mixture process is feasible if and only if final state curve **NOT** passes under initial state curve.

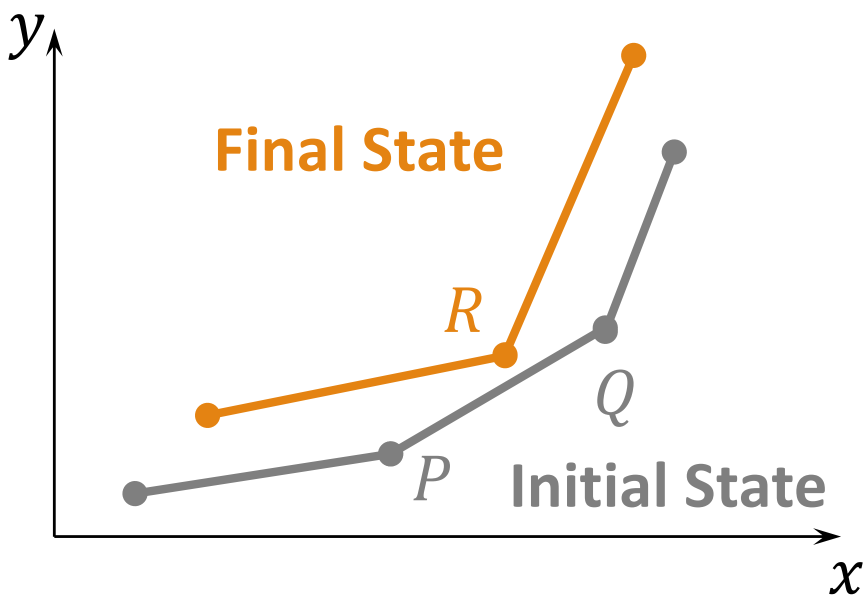




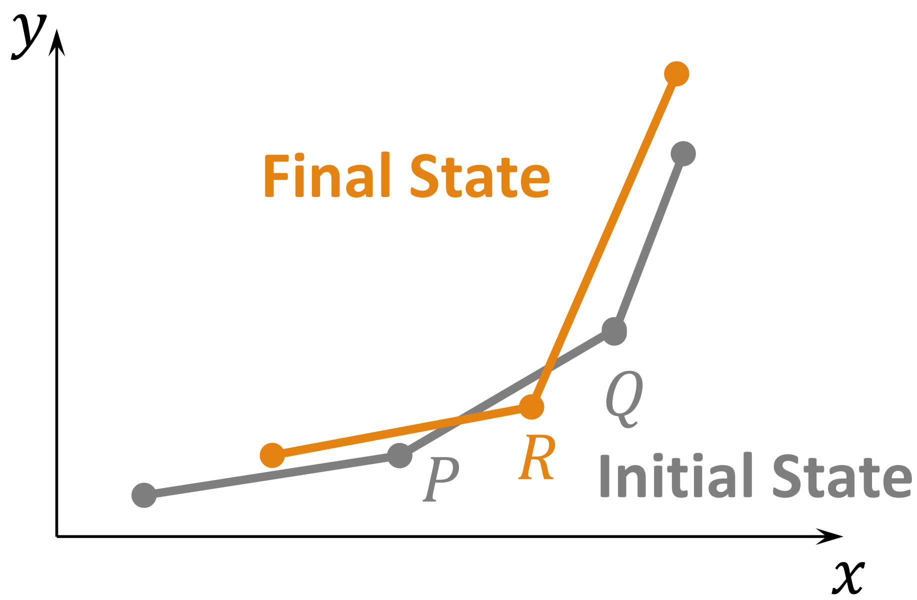
$O(nm)$ Implementation

For each segment PQ of initial curve and each breakpoint R of final curve, check sign of $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$\left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)_z \geq 0$ **is required**



$\left(\overrightarrow{PQ} \times \overrightarrow{PR}\right)_z < 0$ **→ NG**

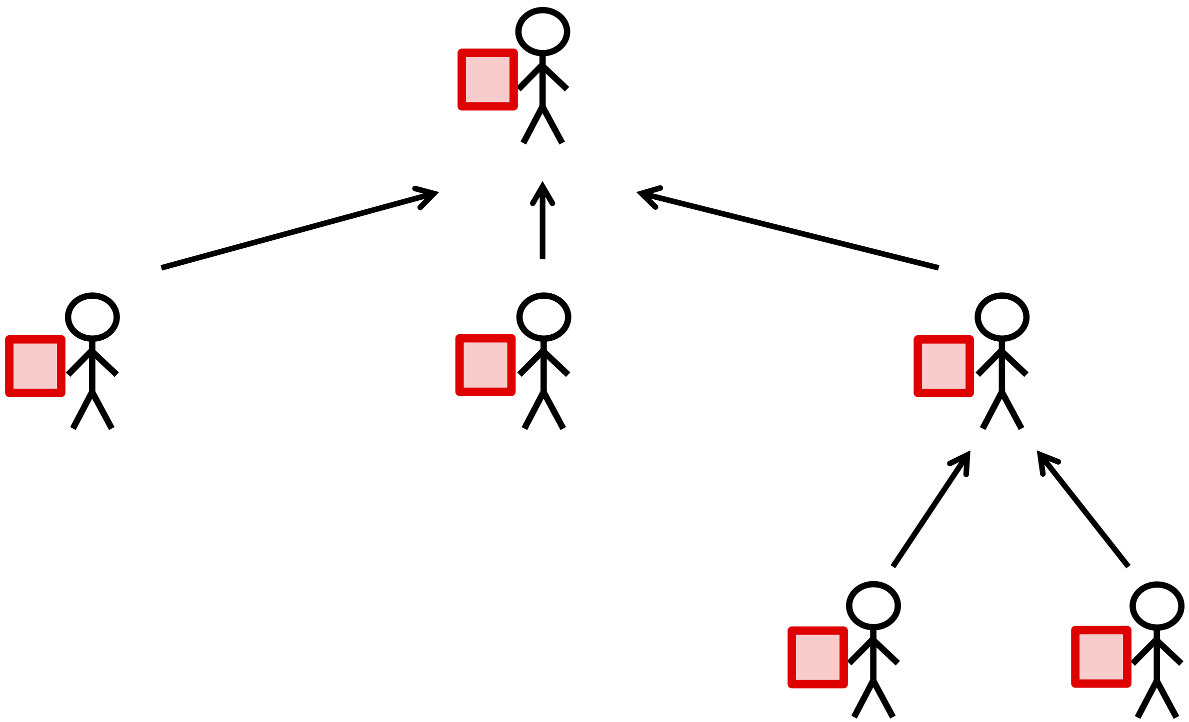


J: Do It Yourself?

PROPOSER: YUTARO YAMAGUCHI
AUTHOR: YUTARO YAMAGUCHI

Story

Complete **the tasks** with **the smallest total fatigue** of employees.



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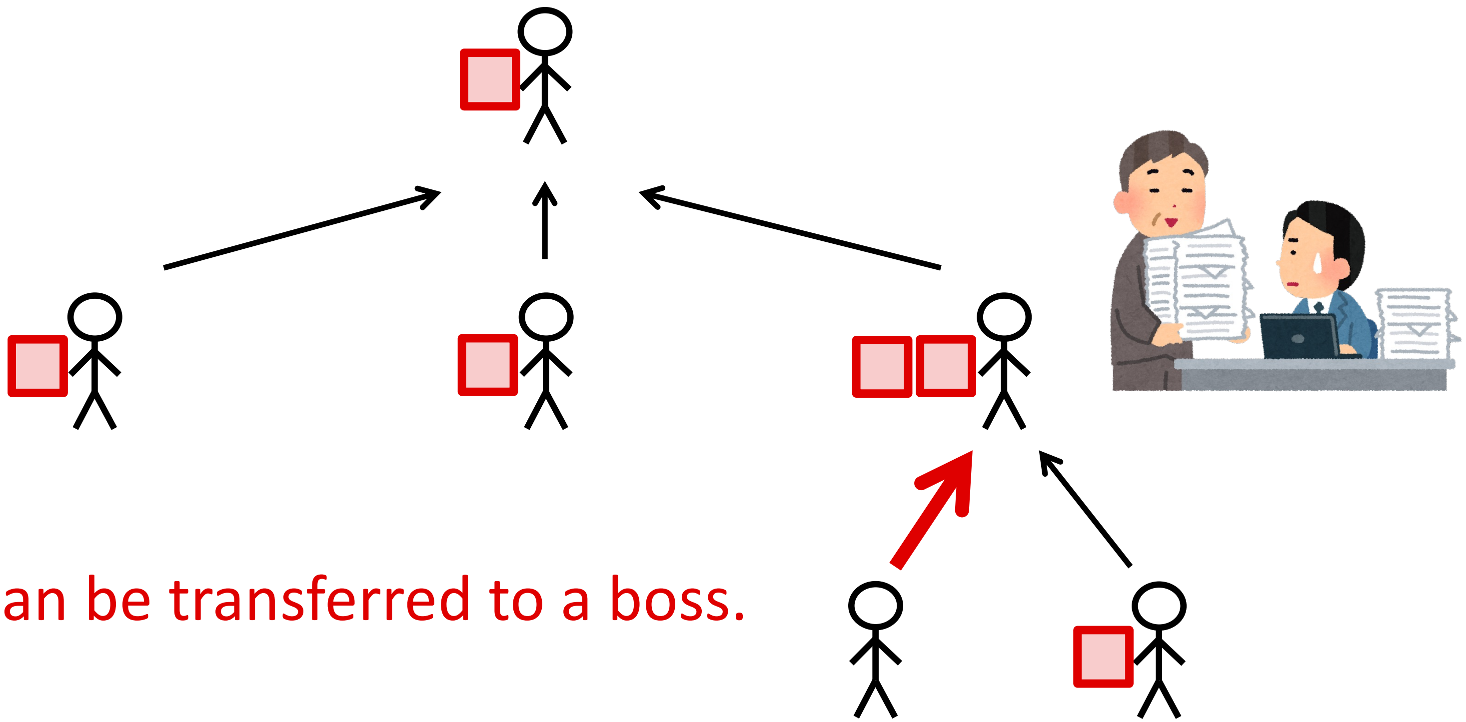

Every task can be transferred to a boss.

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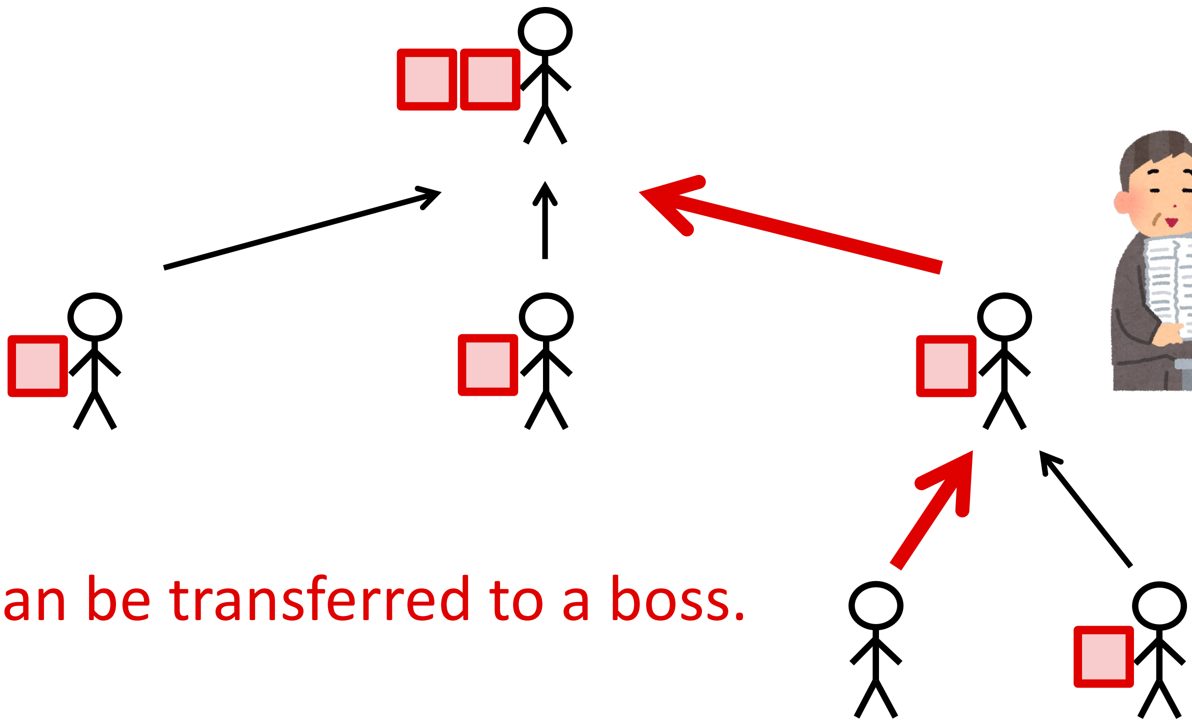

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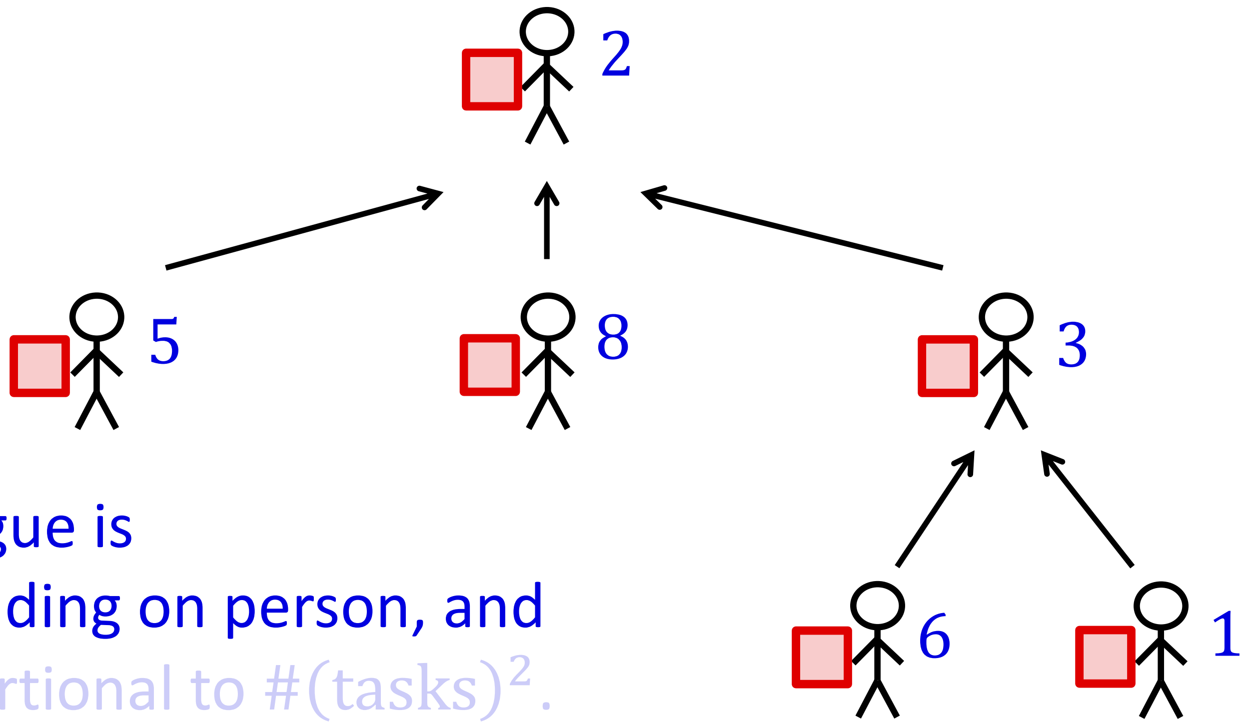


The fatigue is

- depending on person, and
- proportional to $\#(\text{tasks})^2$.

Story

Complete **the tasks** with **the smallest total fatigue** of employees.

$2 \times 5^2 = 50$

$5 \times 0^2 = 0$ $8 \times 0^2 = 0$ $3 \times 1^2 = 3$

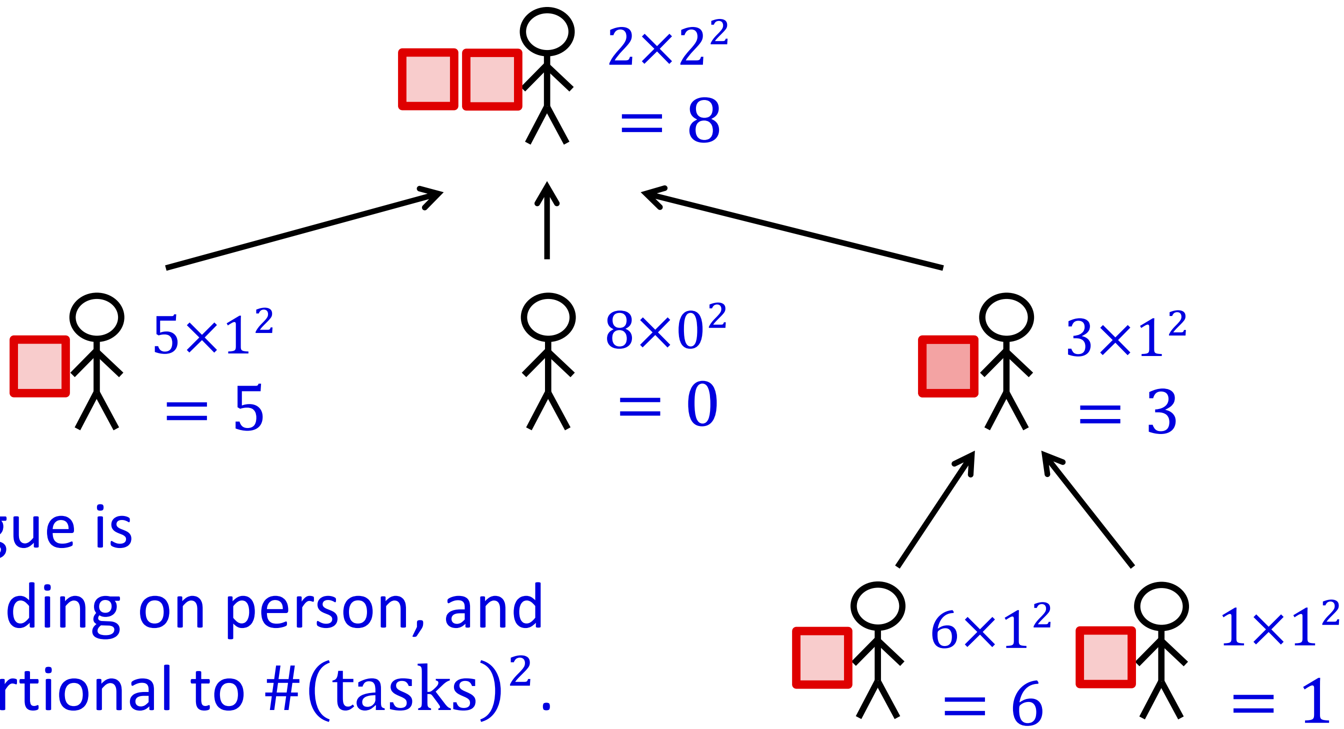
$6 \times 0^2 = 0$ $1 \times 0^2 = 0$

The fatigue is
- depending on person, and
- proportional to $\#(\text{tasks})^2$.

Story

Complete **the tasks** with **the smallest total fatigue** of employees.

$$8+5+0+3+6+1=23$$



The fatigue is

- depending on person, and
- proportional to $\#(\text{tasks})^2$.

Problem

Given a rooted tree of n vertices. $(2 \leq n \leq 5\times10^5)$

Given a fatigability constant f_i of each employee. $(1 \leq f_i \leq 10^{12})$

$$\boxed{\text{minimize } \sum_{i=1}^{n} f_i x_i^2 \text{, where } x_i = \#(\text{tasks done by } \#i)}$$

Solutions

$2 \le n \le 5 \times 10^5$

$1 \le f_i \le 10^{12}$

TL: 10 sec

[AC1] Greedy Algorithm with Heavy-Light Decomposition

- Min-weight base of a laminar matroid (Minimization of M-convex function)
- $O(n \cdot (\log n)^2)$ time

[AC2] Greedy + DP with Weighted-Union Heuristic

- $\mathrm{dp}(v) =$ opt. solution of the subtree of v (maintained by priority queue)
- $O(n \cdot \log n \cdot \log F)$ time $\left(F = \max_i f_i\right)$

[TLE] Naive DP on Tree

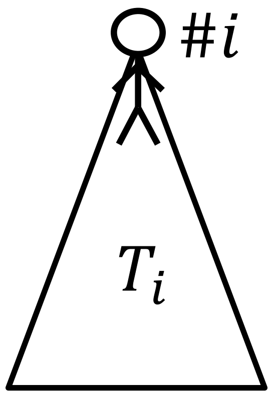
- $\mathrm{dp}(v, k) =$ opt. value of the subtree of v with k tasks completed
- $\Theta(n^2)$ time

Key Observations

$$\text{minimize } \sum_{i=1}^{n} f_i x_i^2 \text{, where } x_i = \#(\text{tasks done by } \#i)$$

- $(x_1, x_2, \dots, x_n)$ is feasible $\iff \sum_{j \in T_i} x_j \le |T_i| \ (\forall i)$,

  where T_i is the subtree of i.



- $f_i x_i^2 = \sum_{k=1}^{x_i} (2k-1) f_i$

  $\rightarrow$ the k-th task takes cost $(2k-1)f_i$

Key Observations

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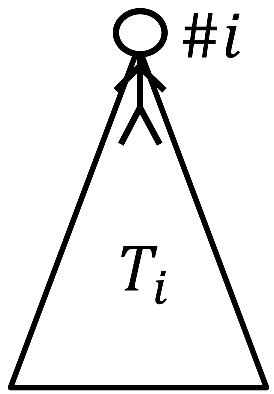

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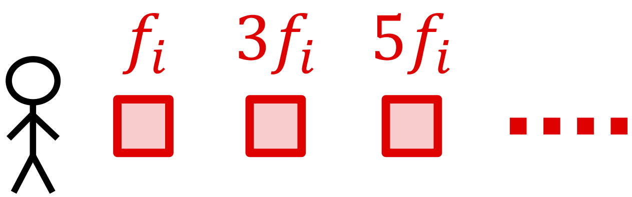


Reformulation

$$\text{minimize } \sum_{i=1}^{n} f_i x_i^2 \text{, where } x_i = \#(\text{tasks done by } \#i)$$

- Each employee $\#i$ has n items with cost $f_i, 3f_i, \ldots, (2n-1)f_i$.
- Minimize the total cost by selecting exactly n items in total subject to at most $|T_i|$ items are selected in each subtree T_i.

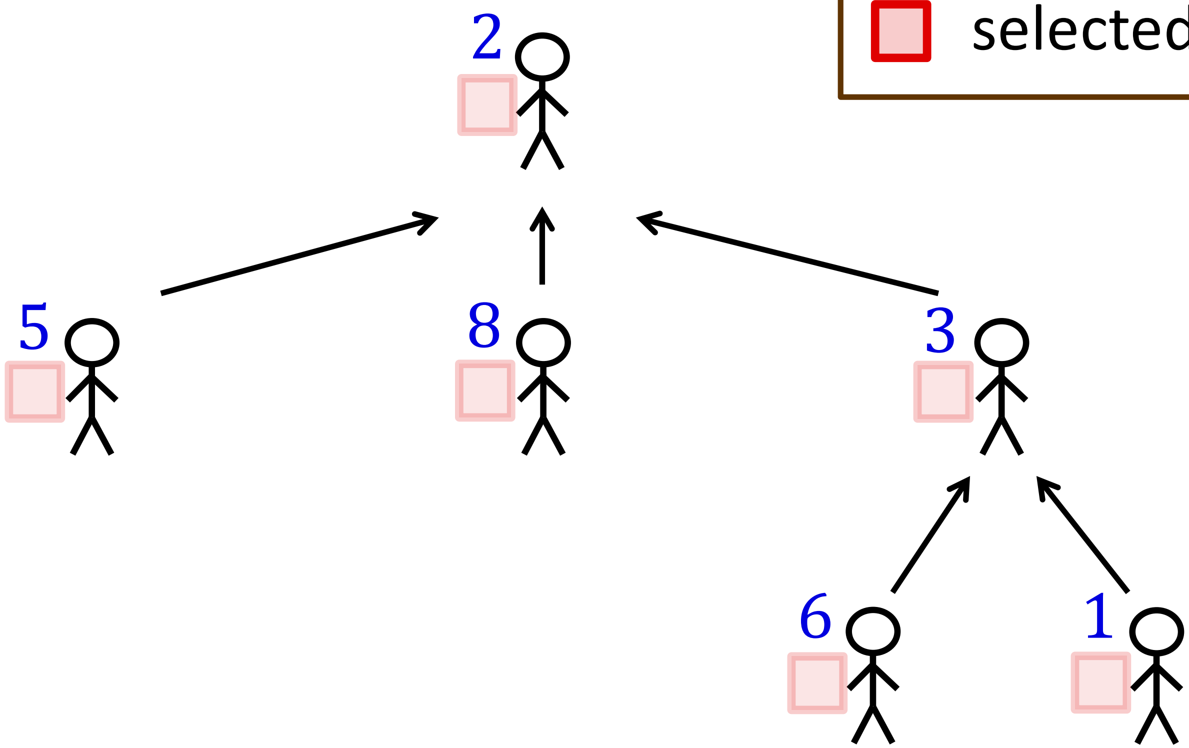
Minimum Weight Base of a Laminar Matroid

$\rightarrow$ Greedy is Optimal

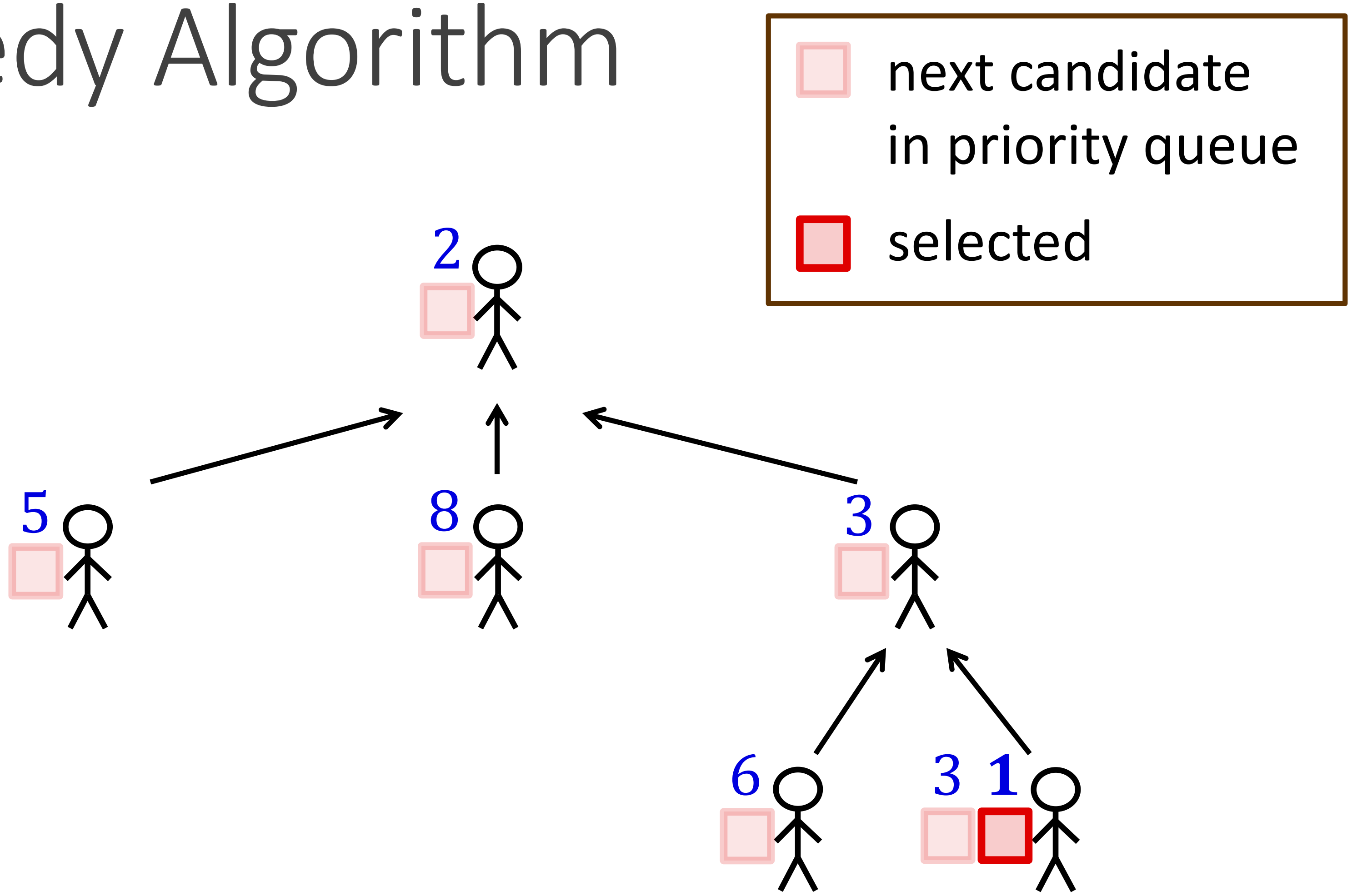
Greedy Algorithm

next candidate in priority queue

selected



Greedy Algorithm



Greedy Algorithm

next candidate in priority queue

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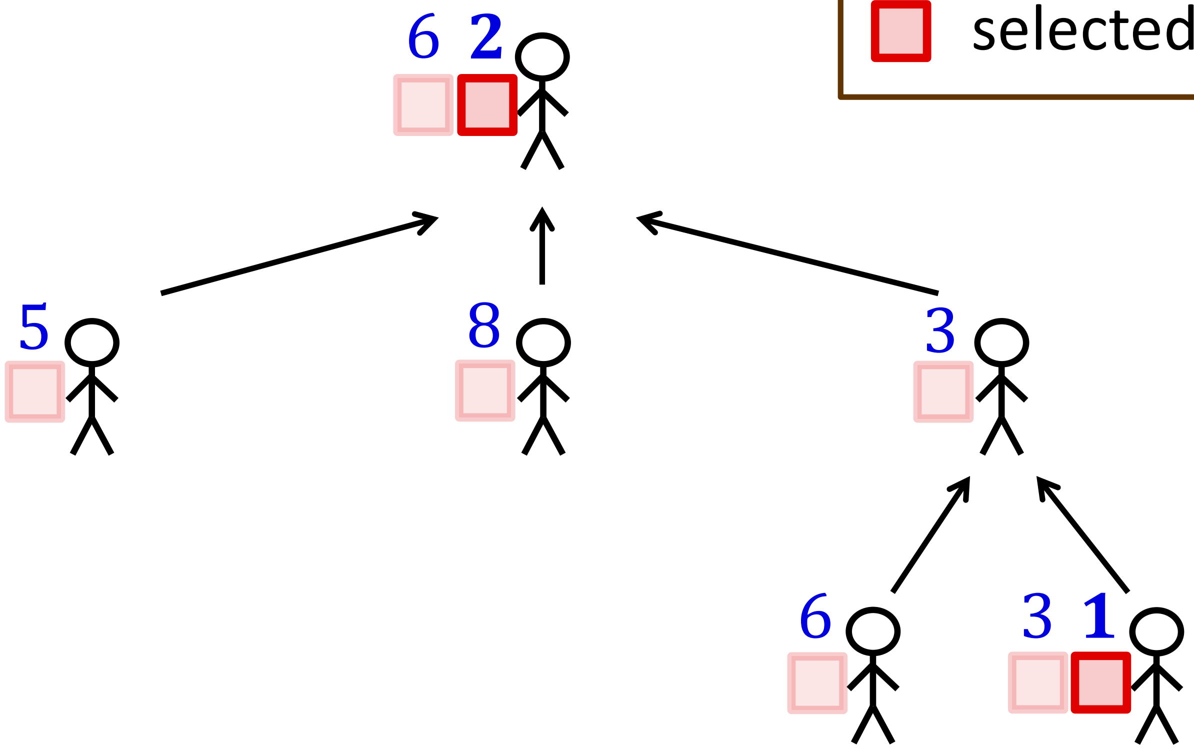

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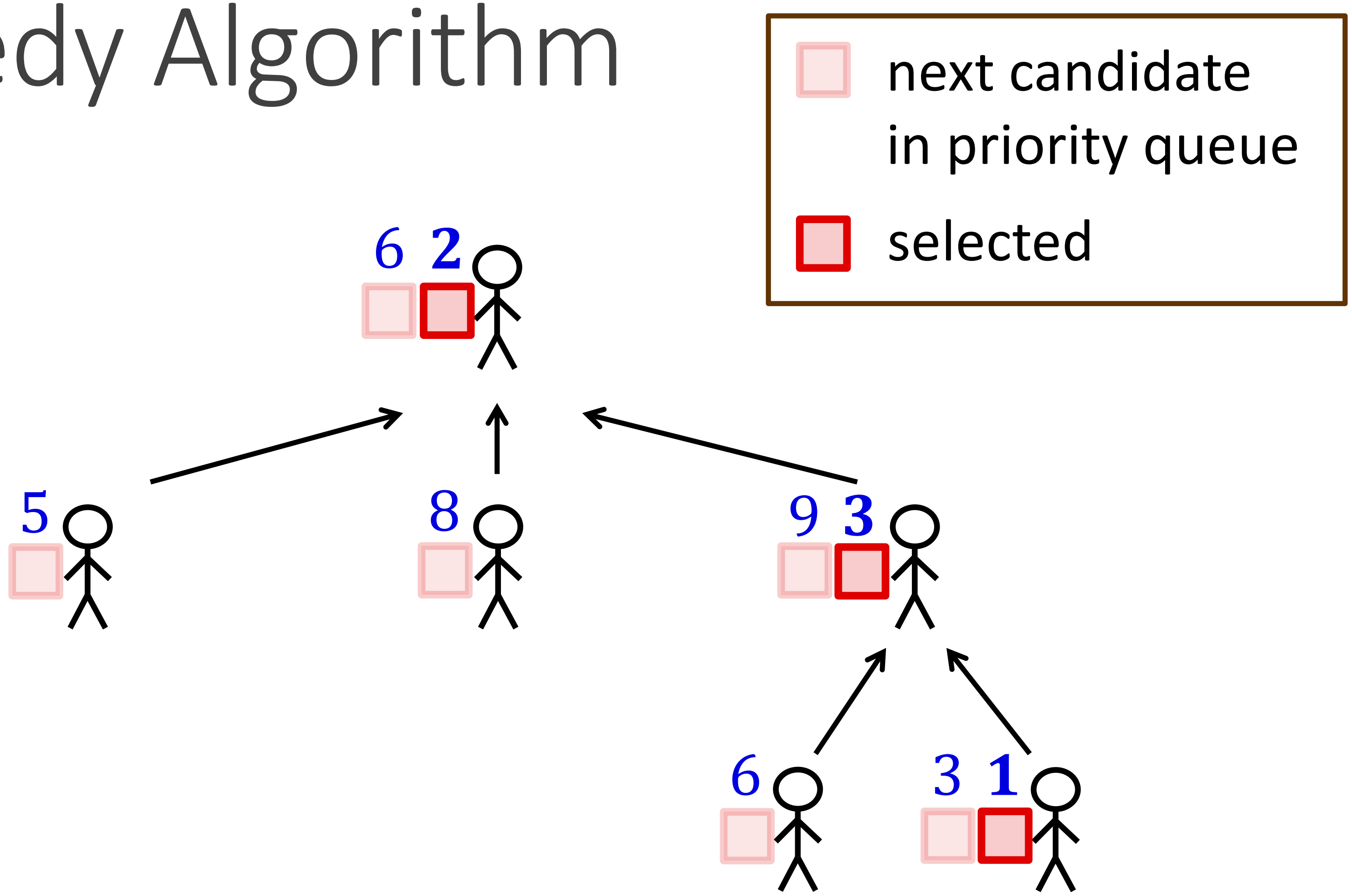
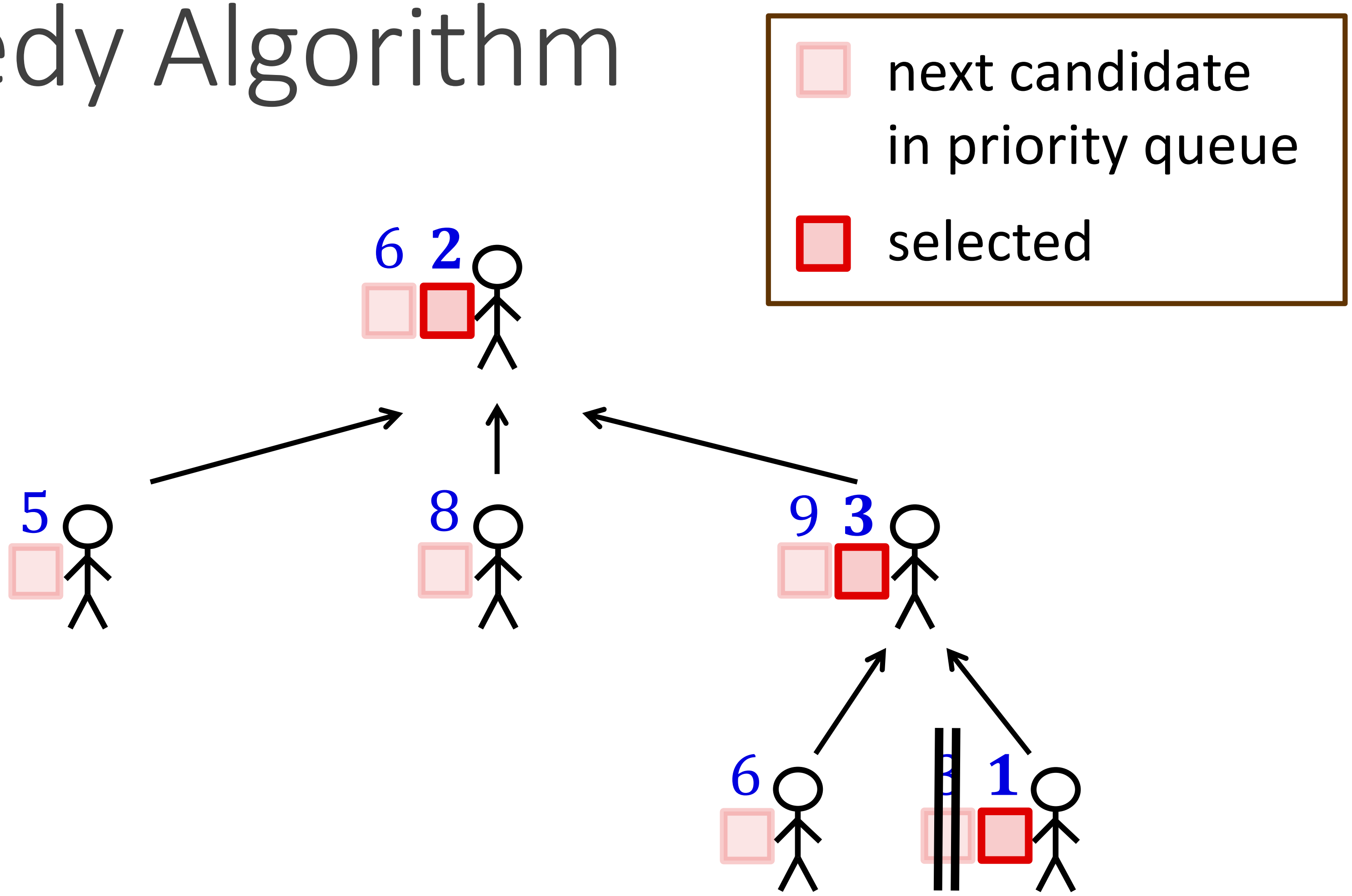


Greedy Algorithm



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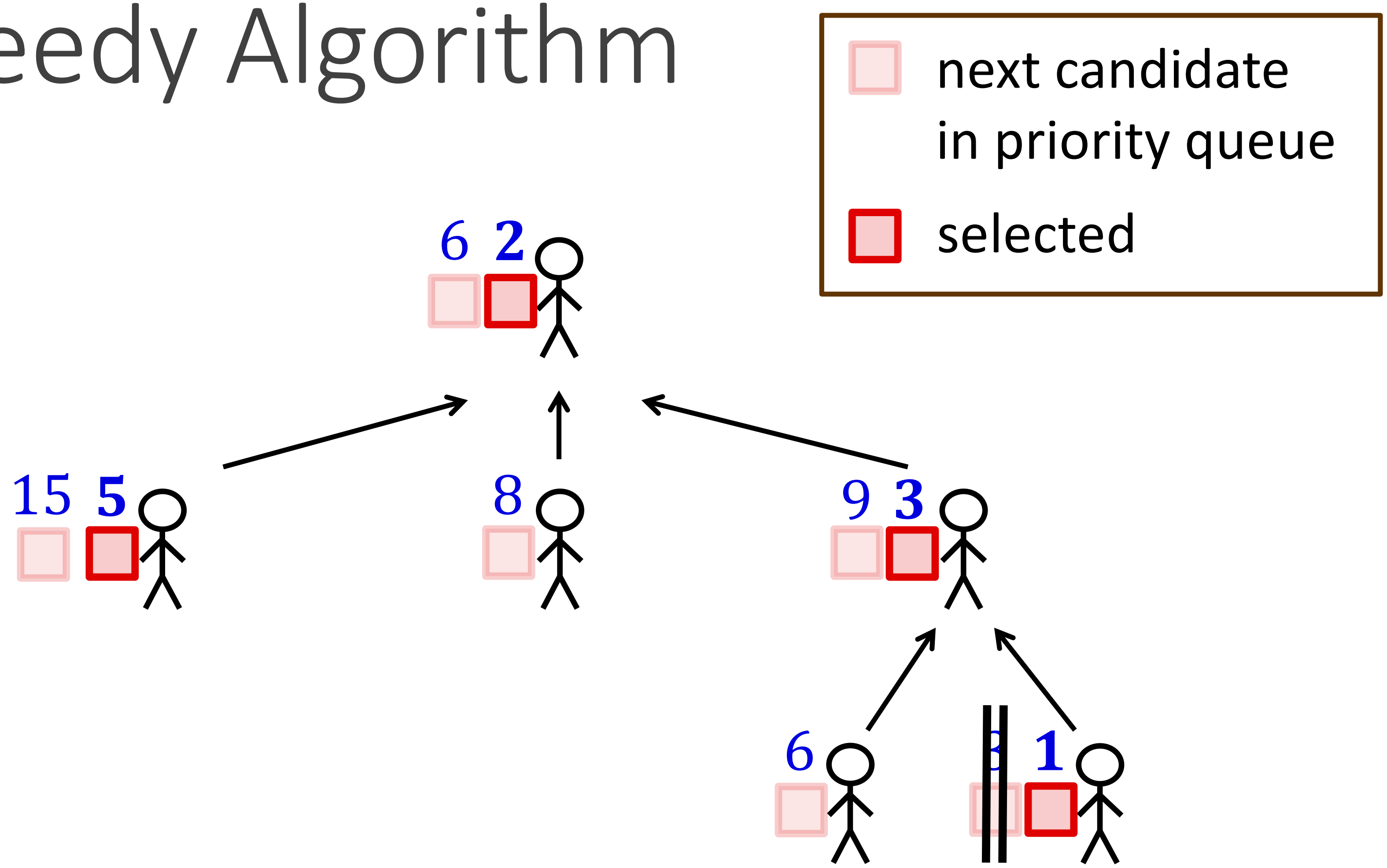
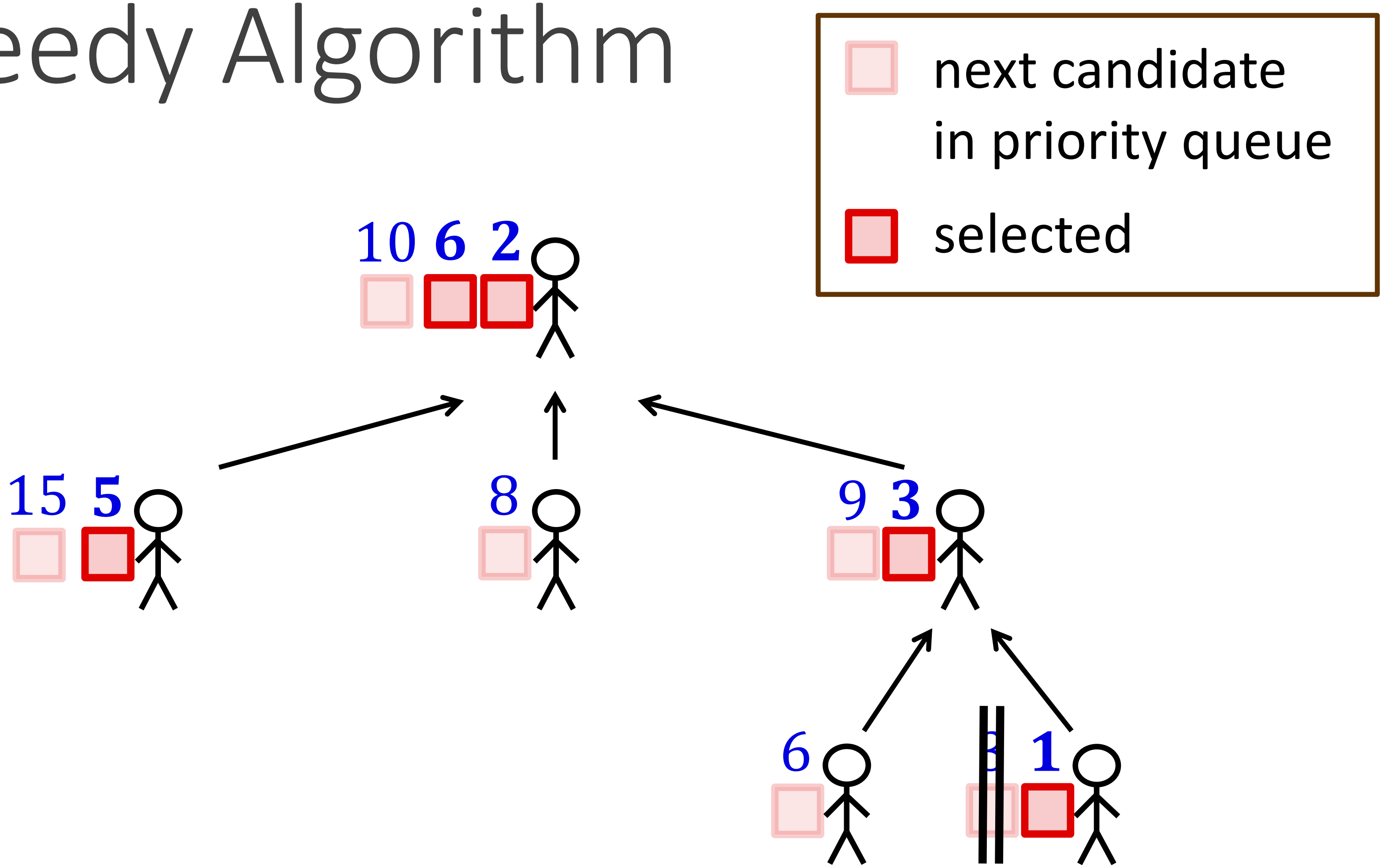
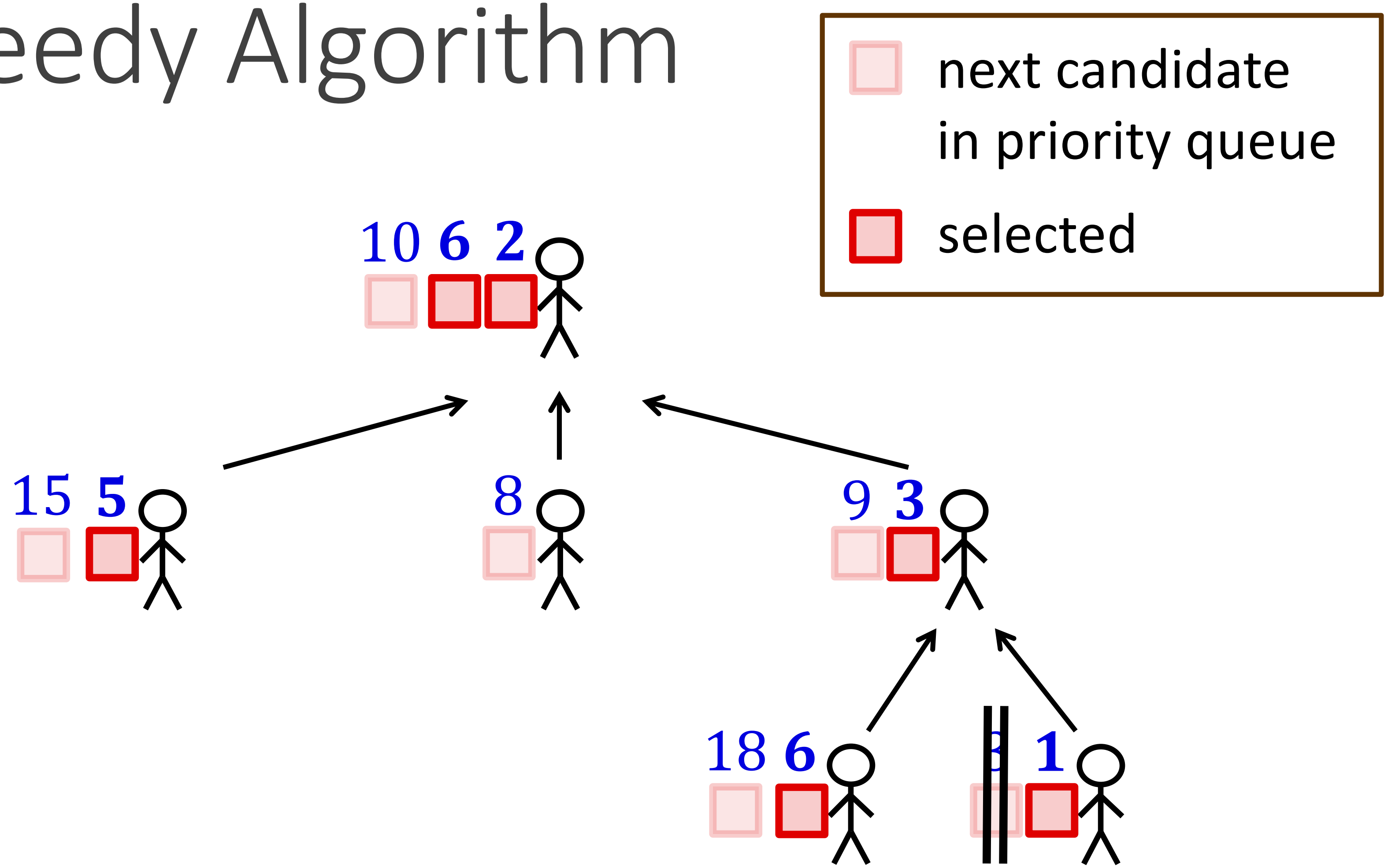


Greedy Algorithm



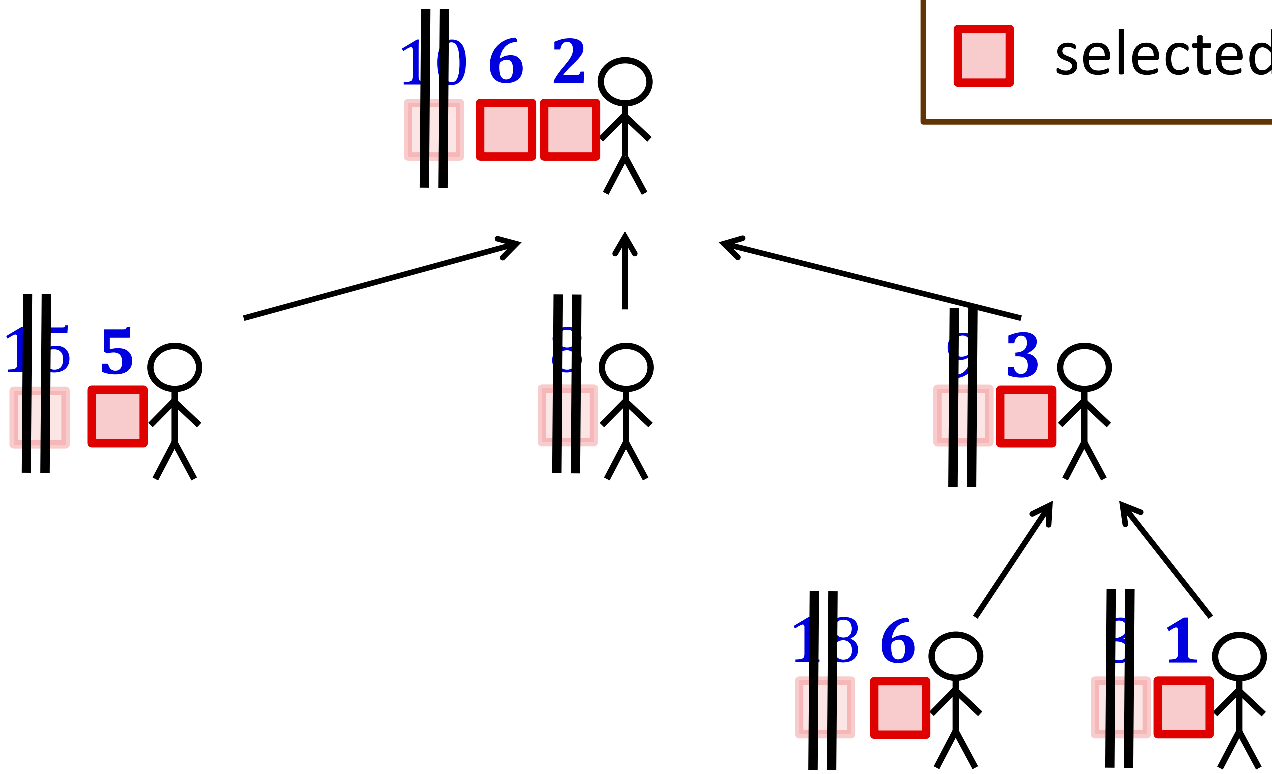
Greedy Algorithm



Greedy Algorithm

next candidate in priority queue

selected



Greedy with HL Decomposition

- An item can be selected

  $\Longleftrightarrow$ The subtree of every boss $\#i$ has positive capacity, i.e.,

  $$\mathrm{cap}(i) \coloneqq |T_i| - \#(\text{items selected in } T_i) > 0$$

- An item is selected $\rightarrow$ Decrease $\mathrm{cap}(i)$ by 1 for every boss $\#i$

- An item is not selected $\rightarrow$ The same person will never work

Range Minimum + **Range Add** $2n$ times

$\mathrm{O}(n \cdot (\log n)^2)$ time with **Heavy-Light Decomposition**

$2 \le n \le 5 \times 10^5$
$1 \le f_i \le 10^{12}$
TL: 10 sec

Solutions

[AC1] Greedy Algorithm with Heavy-Light Decomposition

[AC2] Greedy + DP with Weighted-Union Heuristic

- $\mathrm{dp}(v) =$ opt. solution of the subtree of v (maintained by priority queue)
- $\mathrm{O}(n \cdot \log n \cdot \log F)$ time $\left(F = \max_i f_i\right)$
  - Merge is completed in $\mathrm{O}(n \cdot \log n)$ time (meldable heap) in total;
    $\mathrm{O}(n \cdot (\log n)^2)$ time (usual heap) is also enough.
  - $\#(\text{insertion}) = \mathrm{O}(n \cdot \log F)$ is proved by considering a potential function
    $$\Phi(v) := \sum_{x \in \mathrm{dp}(v)} \log x\,.$$

[TLE] Naive DP on Tree

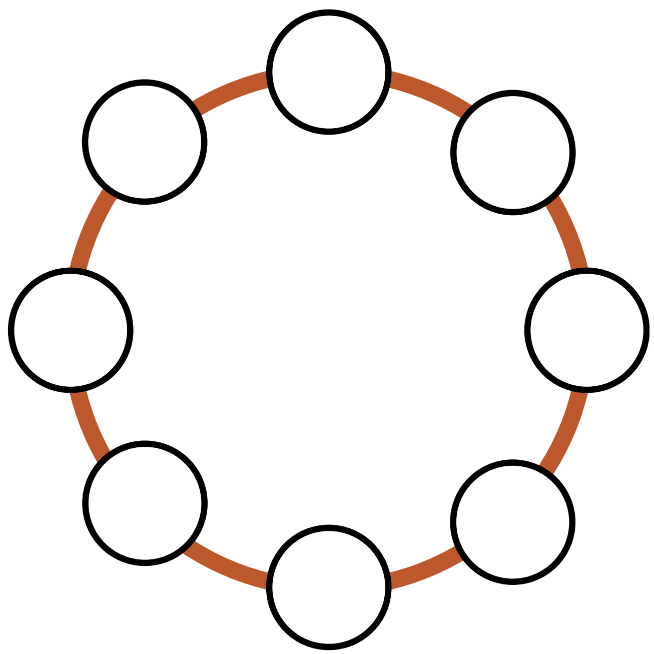
C: Ferris Wheel

PROPOSER: SOH KUMABE

AUTHOR: SOH KUMABE

Problem Description

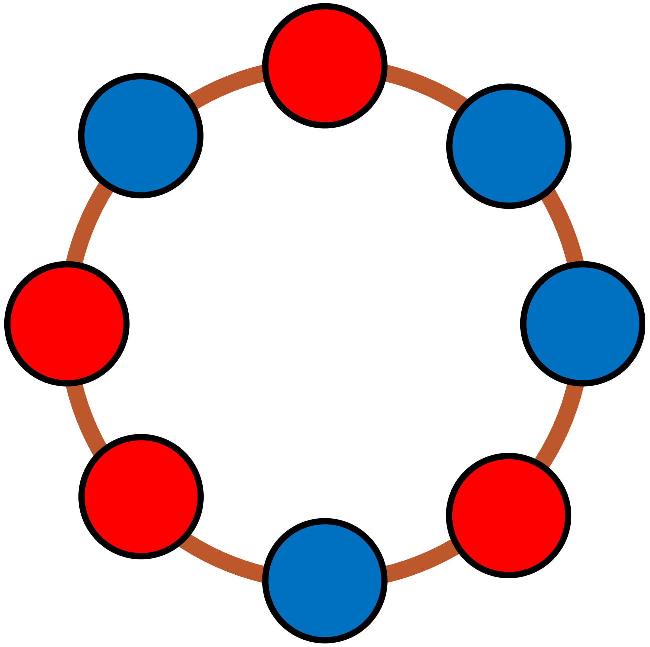
Given $2n$ **points on circle,**



Problem Description

Given $2n$ points on circle,

Count the number of ways to color them by k colors so that



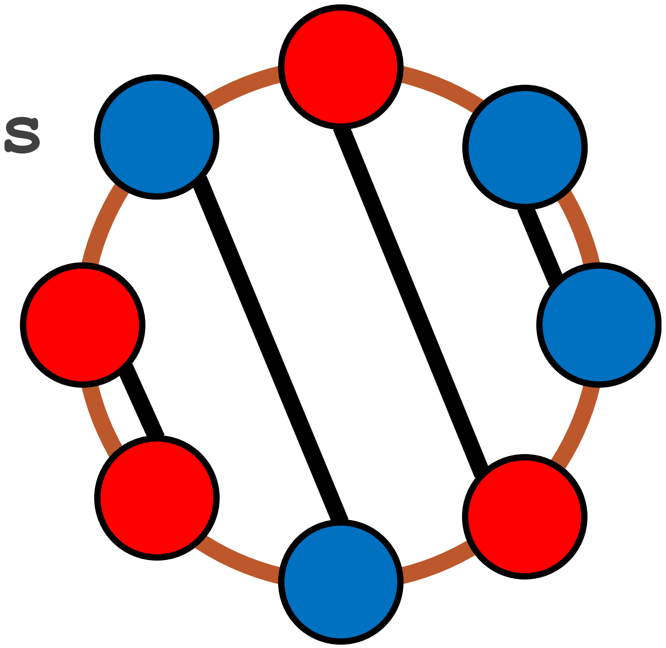
Problem Description

Given $2n$ points on circle,

Count the number of ways to color them by k colors so that

There is a non-crossing perfect matching of points

Such that matched points have the same color



Problem Description

Given $2n$ points on circle,

Count the number of ways to color them by k colors so that

There is a non-crossing perfect matching of points

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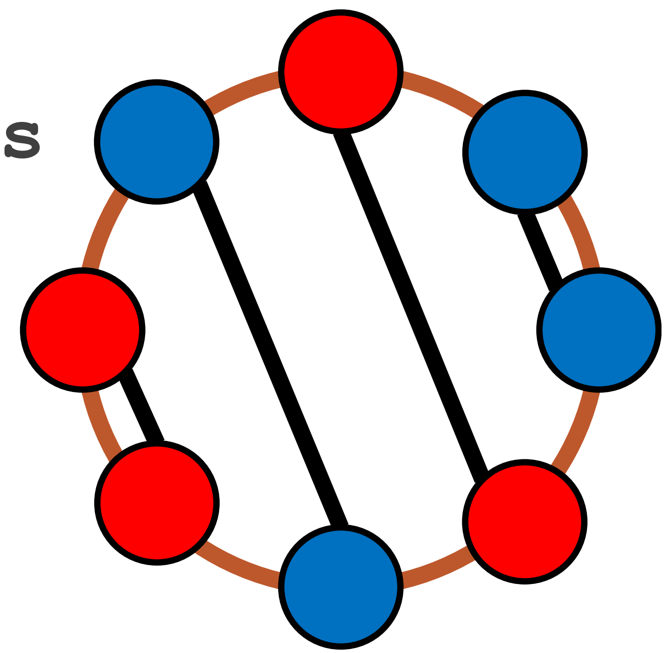
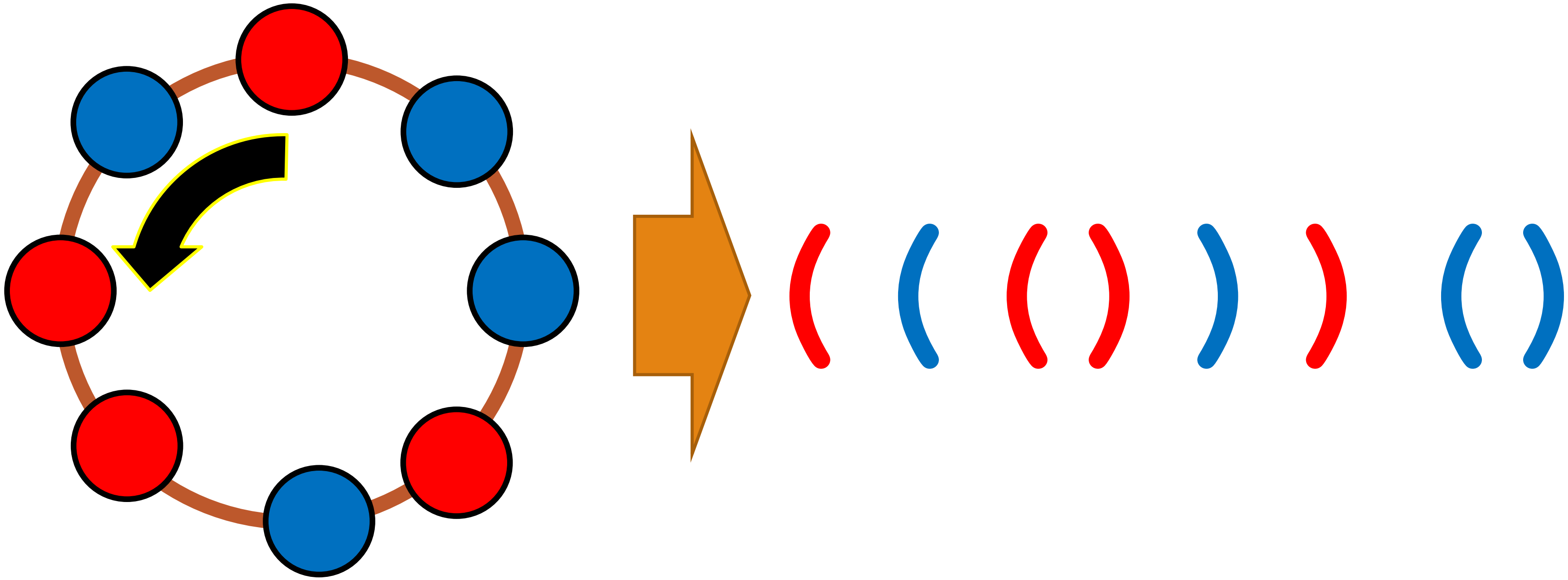
Up to rotation



Matching to Parenthesis

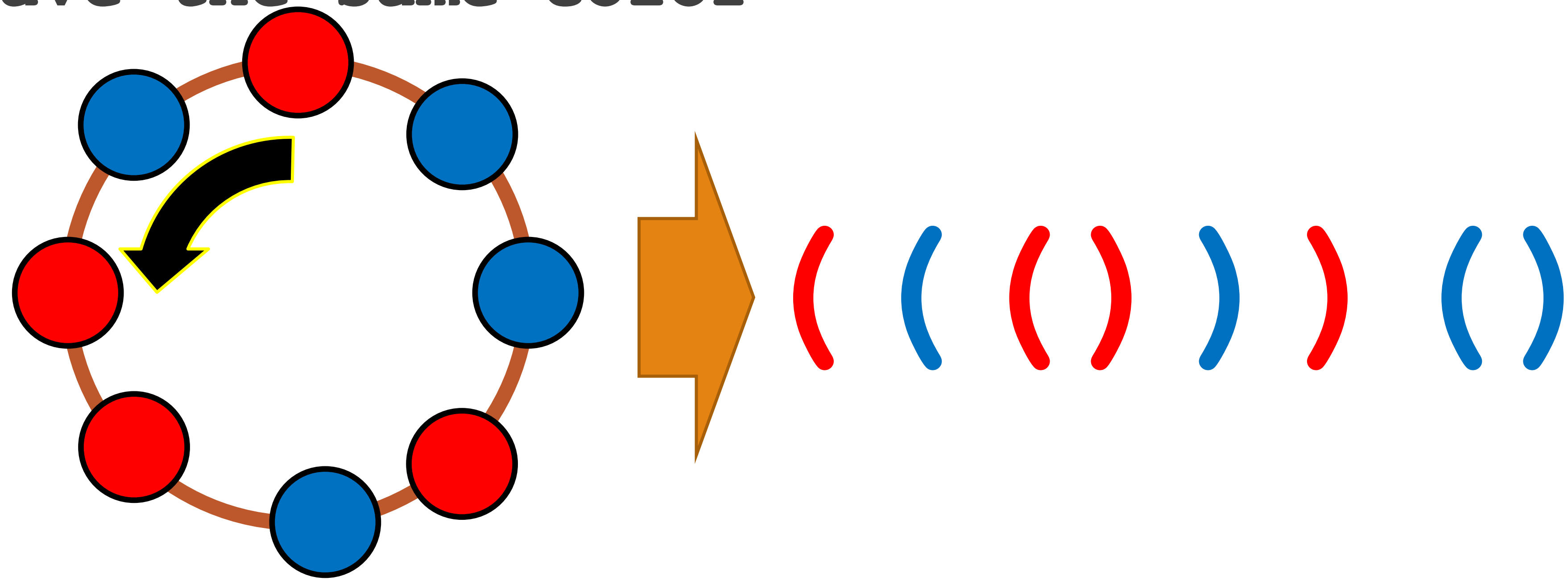


Matching to Parenthesis

There is a non-crossing perfect matching of points

Such that matched points have the same color

Parenthesis is balanced



If not "up to rotation"

Let x_i be the number of balanced parenthesis that have i places with height 0

0 1 2 3 2 1 0 1 0

((( ))) ( )

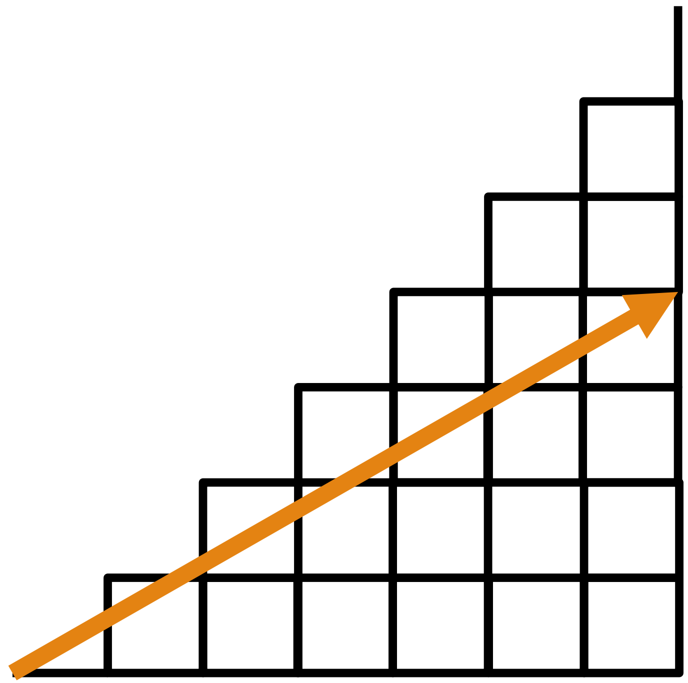
Answer is $\sum_{i=1}^{n} x_i k^i (k-1)^{n-i}$

If not "up to rotation"

Let x_i be the number of balanced parenthesis that have i places with height 0

Can be computed like Catalan numbers

Time Complexity: $O(n)$



“Up to rotation”

Use **Pólya’s enumeration theorem**

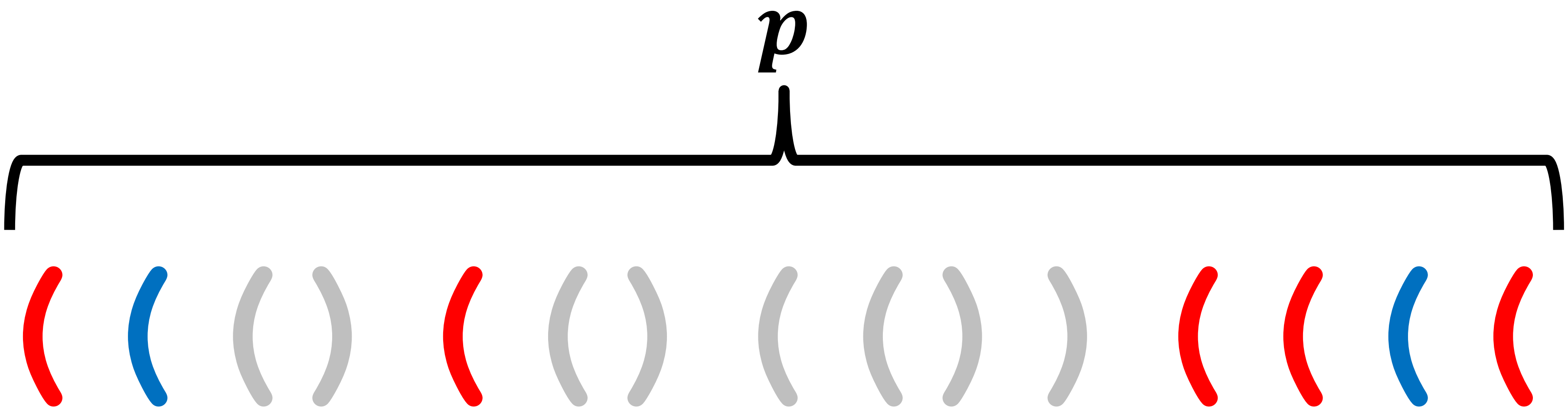
Count **the colorings of period** p

“Up to rotation”

There is a non-crossing perfect matching of points

Such that matched points have the same color

Remaining ‘(‘s are palindrome



“Up to rotation”

If p **is even, no remaining `(`**

same as before

If p **is odd,**
remaining `(`s are palindrome

“Up to rotation”

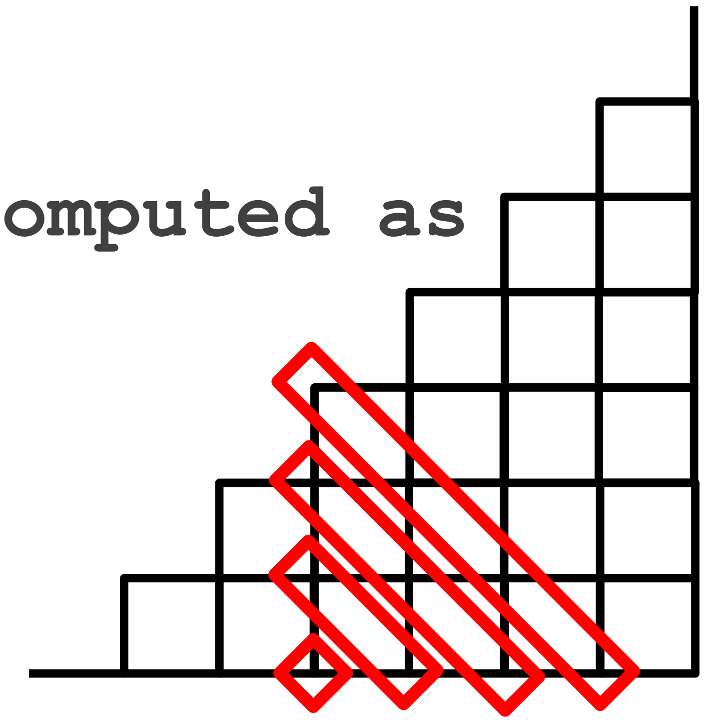
If p is odd,
remaining `(`s are palindrome

Let x_i be the number of parenthesis that have i places with height 0 and some number of `(`s remain

Answer is $\sum_{i=1}^{\frac{p+1}{2}} x_i k^i (k-1)^{\frac{p+1}{2}-i}$

"Up to rotation"

Let x_i be the number of parenthesis that have i places with height 0 and some number of \`(\`s remain



Can be sequentially computed as "diagonal sum" of Catalan number

Time Complexity:
$O(\text{sum of divisors of } 2n)$
$= O(n \log n)$